



Middleton Primary School

Calculation Approach

2025-26

Middleton Primary School Calculation Approach

Purpose of the approach

This approach supports our own Middleton maths curriculum which we implement at our school. Progression within each area of calculation is in line with the programme of study in the 2014 National Curriculum. This calculation approach should be used to support children to develop a deep understanding of number and calculation. This approach has been designed to teach children through the use of concrete, pictorial and abstract representations (where appropriate).

- Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
- Pictorial representation - a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example $12 \times 2 = 24$.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations.

Mathematics Mastery

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through a variety of representations. This approach outlines the different calculation strategies that should be taught and used in Year 1 to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Using the approach

This approach should be used in conjunction with the Middleton maths curriculum and progression documents which have been derived from the NCETM White Rose hub resources/curriculum. Teachers will continue to deepen children's understanding of their year group's expectations and methods without moving on to next year group's objectives. Teachers can use a range of resources to ensure that children are able to secure their understanding of mathematical operations and calculations as outlined in the calculation approach below. Children should have an opportunity to progress within the CPA model to achieve year group expectations.

Resources used within the calculation approach

10 frames

Straws/Pipe cleaners

Bead strings (to 20 and 100)

Rekenrek frames

Base 10/Dienes

Place value Grids

Place value Counters

Double-sided counters





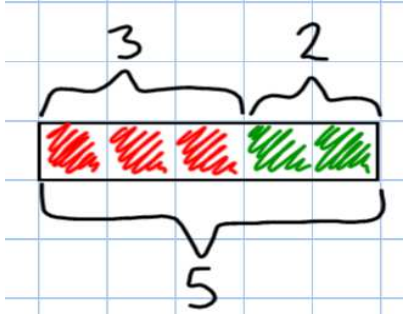
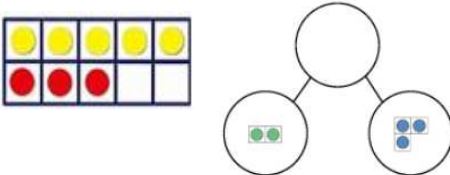
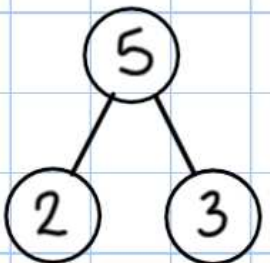

Part-part whole templates

Multi-link cubes

Numicon

Addition

Early Years Foundation Stage

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Knows that a group of things change in quantity when something is added</p> <p>Find the total number of items in two groups by counting all of them.</p> <p>Says the number that is one more than a given number.</p> <p>Finds one more from a group of up to five objects, then ten objects.</p> <p>In practical activities and discussions, beginning to use the vocabulary involved in adding.</p> <p>Using quantities and objects, they add two single digit numbers and count on to find the answer.</p> <p>Solve problems using doubling.</p>	<p>Use toys and general classroom resources for children to physically manipulate, group and regroup.</p>  <p>Use specific maths resources such as counters, numicon, multi-link cubes etc.</p>  <p>Use visual supports such as ten frames, part-part whole models and addition mats with the physical objects and resources that can be manipulated,</p> 	<p>Two groups of pictures so children can count the total number of objects.</p>  <p style="text-align: center;">___ + ___ = ___</p> <p>Bar model using visuals, pictures/icons or colours.</p>  <p>Use visual supports such as ten frames, part part-whole models and addition mats with pictures, icons or marks.</p> 	<p>A focus on symbols and numbers to form a calculation.</p>   <p style="font-size: 2em; text-align: center;">2 + 3 = 5</p> <p>**There are no expectations for the children to be able to record a number sentence/addition calculation in EYFS.</p>

Year 1

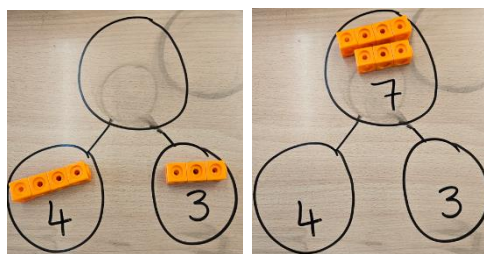
Objectives/Strategy

Concrete

Use multilink cubes to add two numbers together as a group or in a bar. (some children may still need to use real objects)



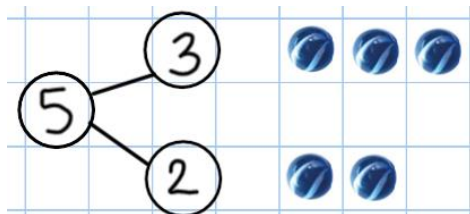
Use part-part whole models with cubes.



Combining two parts to make a whole: part-whole models

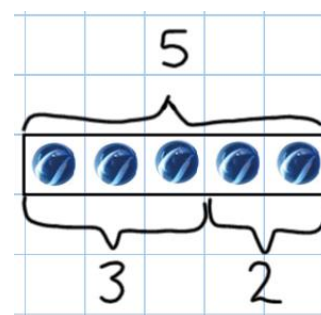
Pictorial

Use pictures to add two numbers as a group.

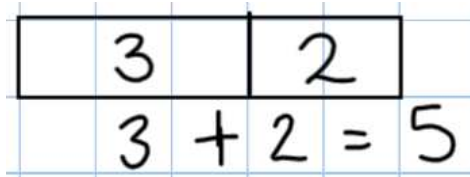


The bar model will be continued from EYFS to support problem solving and bridge the gap between concrete and abstract representations.

Pictorial bar model (with images/objects)

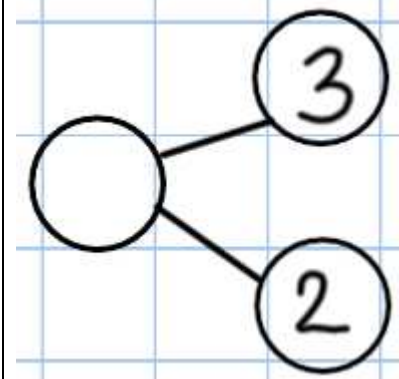


Abstract bar models



Abstract

Use the part-part whole model as shown to move into the abstract.

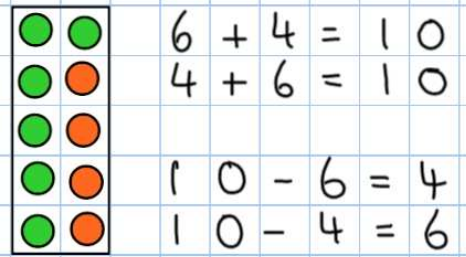


$$6 + 3 = 9$$

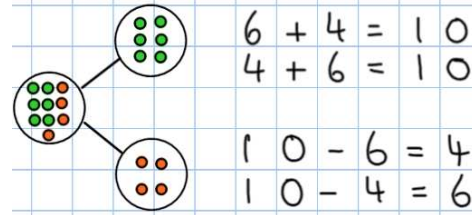
$$4 + 1 = 5$$

Represent and use number bonds and related subtraction facts within 20.

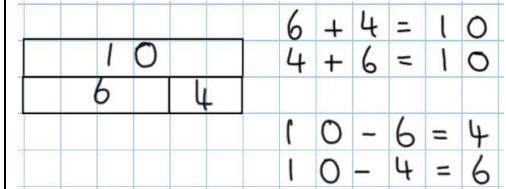
Some children may need to initially use real objects in ten frames then move onto representation.



Part-Part Whole models with concrete representations to explore number bonds and related facts.

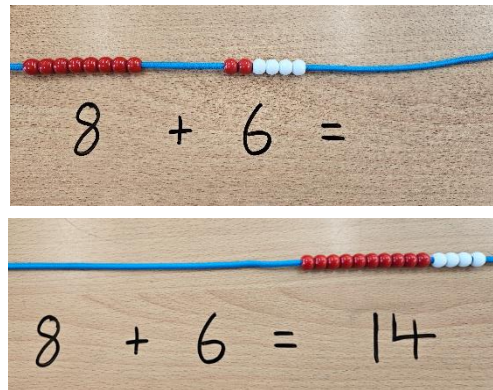


Bar models and part-part whole models to be used alongside the abstract number sentences.

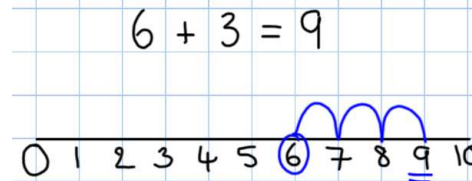


Addition and subtraction of one-digit and two-digit numbers to 20 including 0. Start at the bigger number and counting on

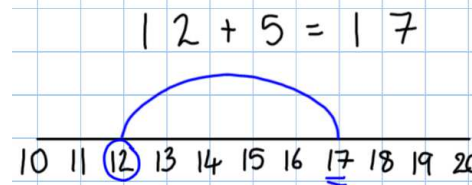
Start with the larger number on the bead string or rekenrek and then count on the smaller number 1 at a time to find the answer.



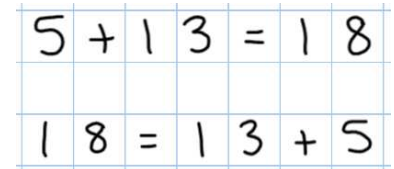
Start at the larger number on the number line and count on in ones.



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

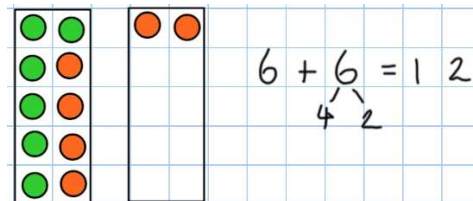


Place the larger number in your head and count on the smaller number to find the answer.



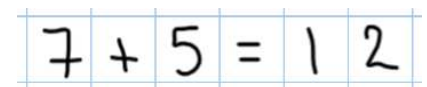
Regrouping to make 10 (The 'Make 10' Strategy).

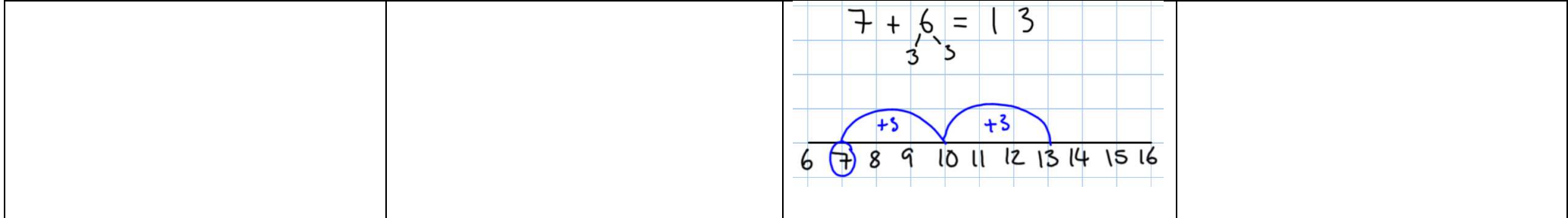
Start with the bigger number and use the smaller number to make 10 use ten frames.



Use a number line. Regroup or partition the smaller number using the part-part whole model to make 10.

If I am at seven how many more do I need to make 12? Then, how many more do I need to add on now?

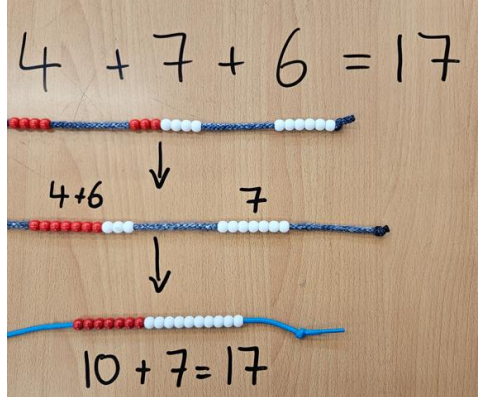
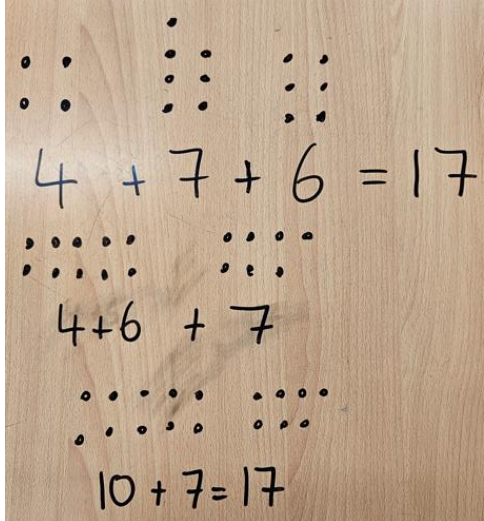
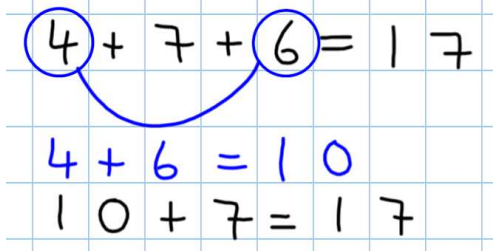
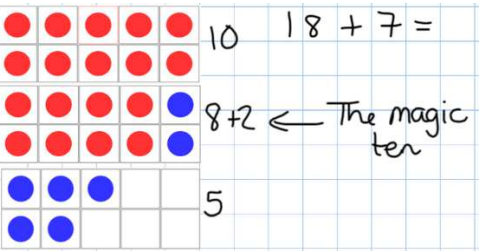
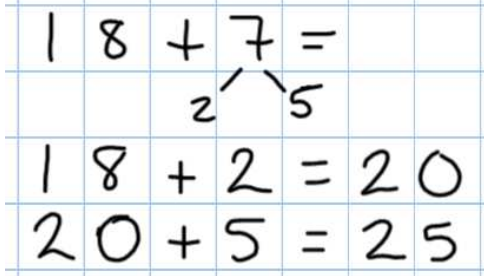
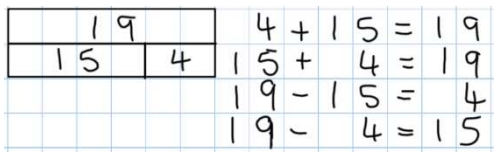




Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part-whole

Year 2

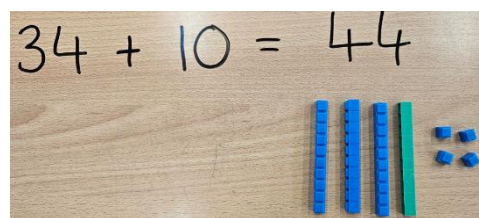
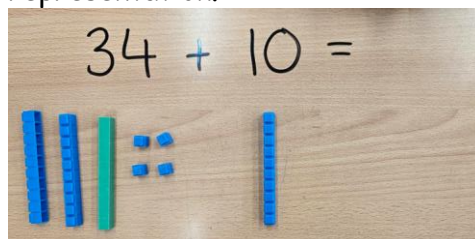
Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Adding 3 one-digit numbers together.</p>	<p>Following on from making 10 in Year 1, make 10 with 2 of the digits (if possible using number bonds to 10) and then, add on the third digit.</p> 	<p>Add together three groups of objects. Draw a picture to recombine the groups to make 10.</p> 	<p>Combine the two numbers that make 10 and then add on the remainder mentally.</p> 
<p>Adding a two-digit number and ones.</p>	<p>Use ten frames to make 'magic' ten.</p> 	<p>Use part part-whole models and bar model to model answer.</p> 	<p>Explore related number facts using a bar model.</p> 

$$15 + 4 = 19$$

	19	
15		4

Adding a 2-digit number and multiples of 10.

Explore that the ones digit does not change when adding ten. Use dienes to aid concrete representation.

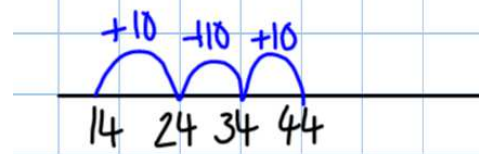


Adding 2 two-digit numbers (no regrouping)

Add together the ones first then add the tens. Use the dienes blocks first before moving on to place value counters.

Base 10 may be used above the number line initially. The calculation needs to be shown above or alongside the number line to see the connection.

$$14 + 30 = 44$$



After practically using the dienes blocks and place value counters to help solve addition problems in place value charts.

Add multiples of 10 to a number mentally.

$$27 + 10 = 37$$

$$27 + 20 = 47$$

$$27 + ? = 57$$

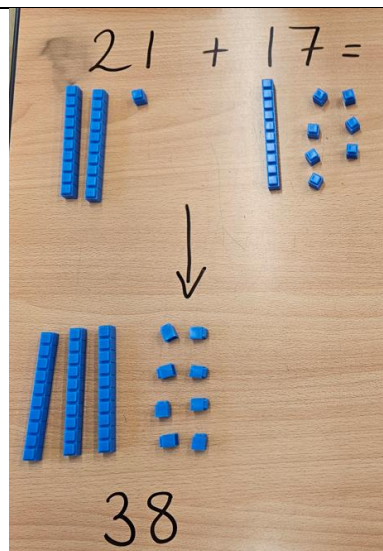
Partition to add method.

$$\begin{array}{r} 21 \\ 20 \end{array} + \begin{array}{r} 17 \\ 10 \end{array} = \begin{array}{r} 38 \\ 30 \end{array}$$

$$20 + 10 = 30$$

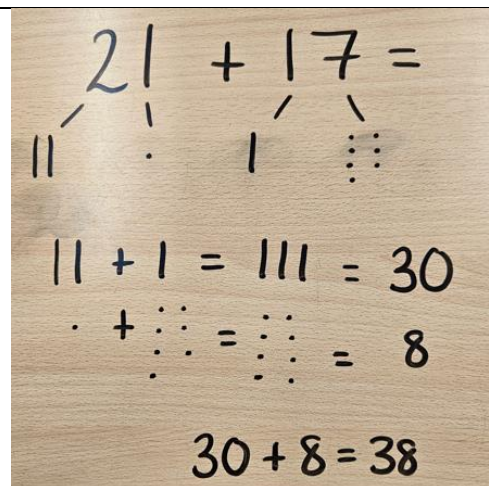
$$1 + 7 = 8$$

$$30 + 8 = 38$$



**Some children may not be ready for place value counters in Year 2.

Numicon can also be used to support understanding.



**Use this pictorial method alongside the partition to add abstract method to help understanding. You could replace the drawings for actual dienes sticks if preferred.

Recording addition in columns supports place value, understanding and presentation of methods. This also prepares children for the formal written methods with larger numbers in KS2.

Children to move to more formal recording using partitioning method:

Adding 2 two-digit numbers (with regrouping)

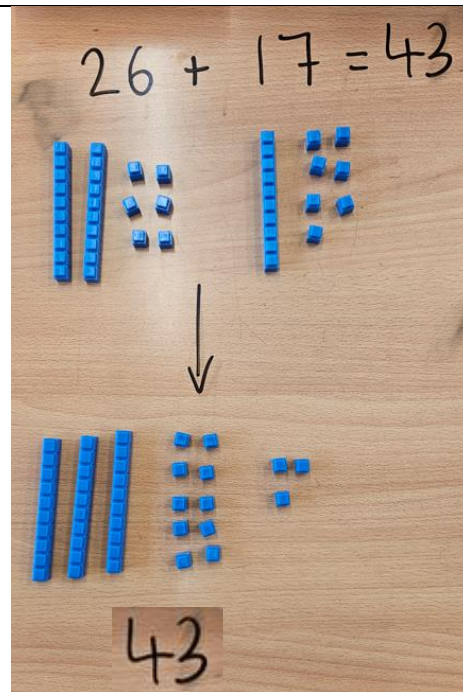
Add together the ones first then add the tens. Use the dienes blocks first before moving on to place value counters.

After practically using the dienes blocks and place value counters to help solve addition problems in place value charts.

Partition to add method.

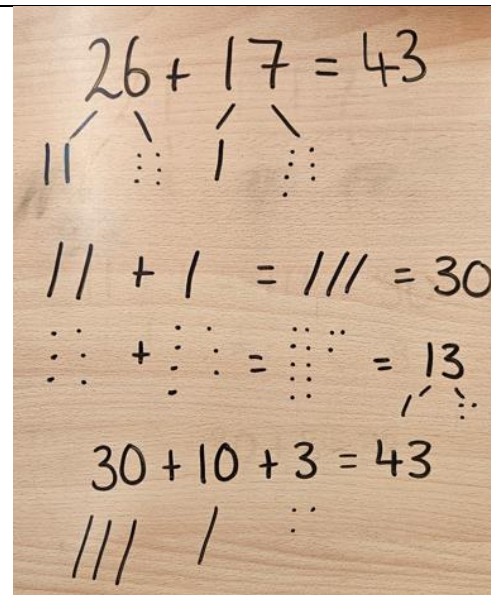
2	6	+	1	7	=	4	3
20	6		10	7			
20		+	10		=	30	
	6	+		7	=	13	
30		+	13		=	43	

Recording addition in columns supports place value, understanding and presentation of methods. This also prepares children for the



**Some children may not be ready for place value counters in Year 2.

Numicon can also be used to support understanding.



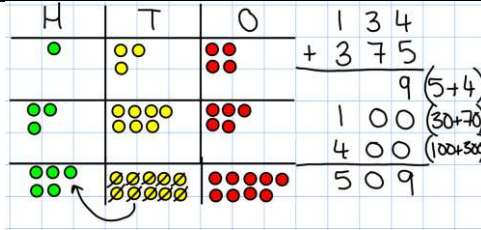
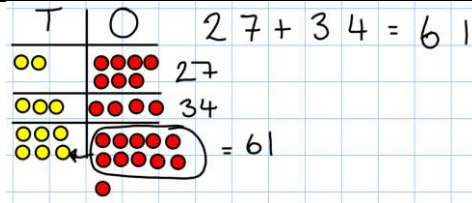
**Use this pictorial method alongside the partition to add abstract method to help understanding. You could replace the drawings for actual dienes sticks if preferred.

formal written methods with larger numbers in KS2.

Children to move to more formal recording using partitioning method:

Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens, boundary



$$\begin{array}{r} 21 + 17 = 38 \\ \underline{20 \quad 1} \quad \underline{10 \quad 7} \end{array}$$

$$\begin{array}{r} 20 + 10 = 30 \\ 1 + 7 = 8 \\ \hline 30 + 8 = 38 \end{array}$$

Once children are secure with partition to add method, the method below should be used to support the transition to a more formal written method.

H	T	O	
2	3	5	
+ 3	8	4	
		9	(5 + 4)
1	1	0	(30 + 80)
5	0	0	(200 + 300)
<u>6</u>	<u>1</u>	<u>9</u>	

Vocabulary

addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Year 4

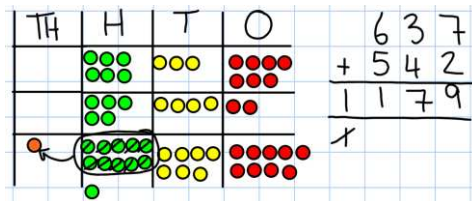
Objectives/Strategy

Concrete

Pictorial

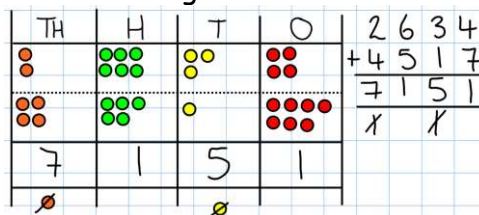
Abstract

Children continue to use dienes or place value counters to add, exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.



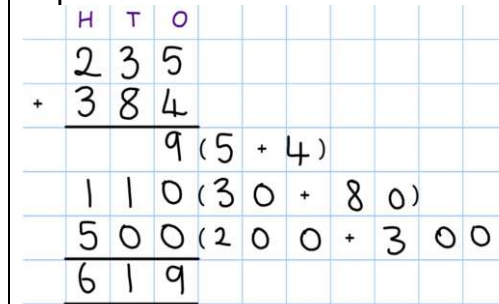
**The calculation must be shown above or alongside the manipulatives to see the connections between them.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.

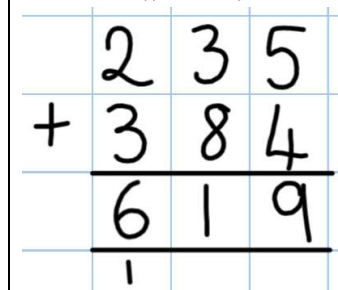


Using formal written methods of compact column addition, where appropriate, for numbers with up to four-digits (with and without exchange)

Recap previous Year 3 method of expanded column addition.



Once children secure with the expanded method, convert to the compact column addition method as shown below as the new formal written method.

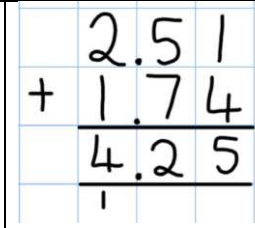
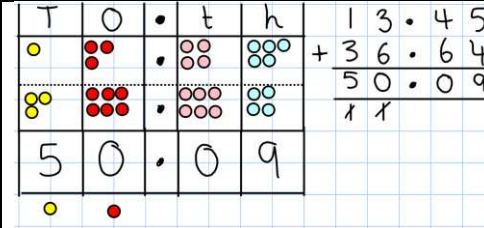
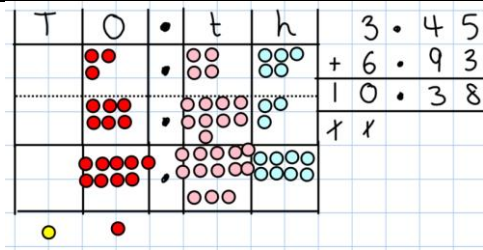


Add decimals with 2 decimal places, including money

Introduce decimal place value counters and model exchange for addition.

Show in pictorial form, exchanging hundredths for tenths and tenths for 1s if necessary.

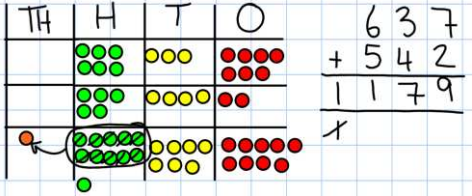
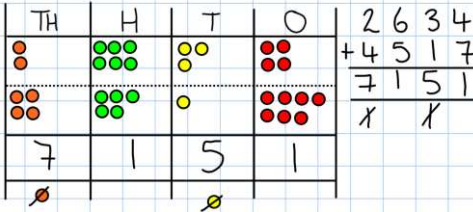
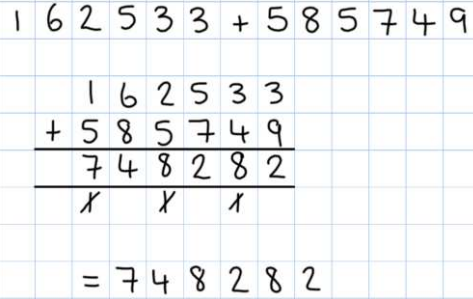
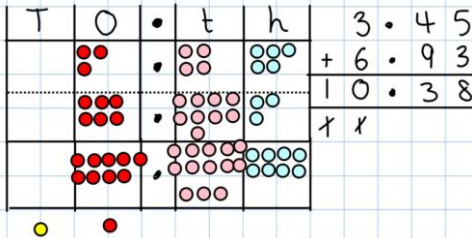
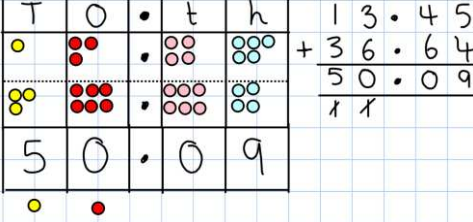
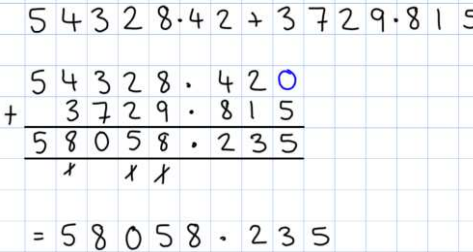
As the children move on, introduce decimals with the same number of decimal places. Money can be used here as a route into the learning.



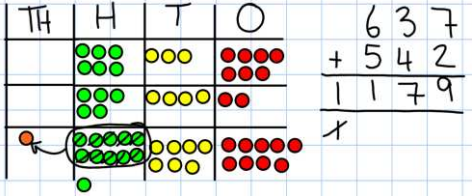
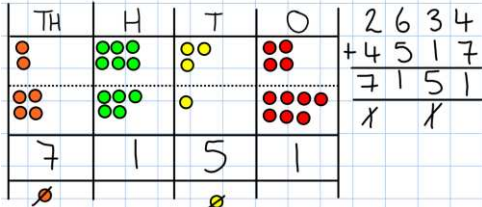
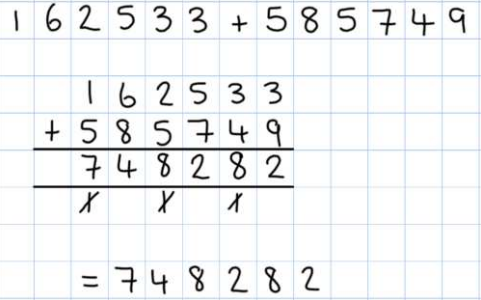
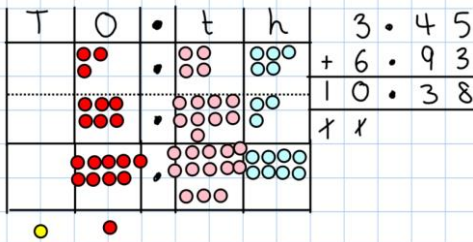
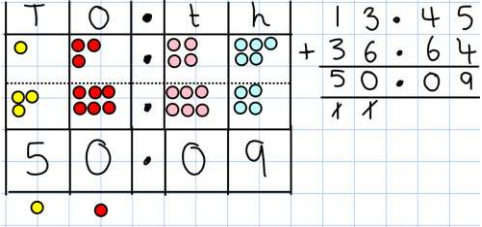
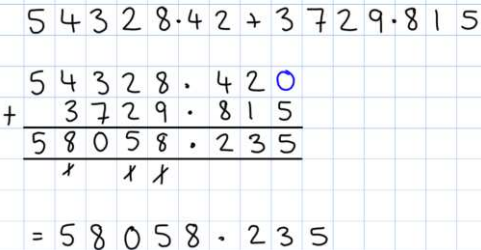
Vocabulary

addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point

Year 5

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Add numbers with more than four-digits up to one million using the compact column addition method with and without exchanging.</p>	<p>Children continue to use dienes or place value counters to add, exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Children should have abstract supported by a pictorial or concrete if needed.</p> 
<p>Add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points (up to 3dp).</p>	<p>Introduce decimal place value counters and model exchange for addition.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Show in pictorial form, exchanging hundredths for tenths and tenths to 1s if necessary.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Insert zeros for place holders when solving decimal problems where the addends do not have the same number of decimal places.</p> 
<p>Vocabulary</p>	<p>addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point</p>		




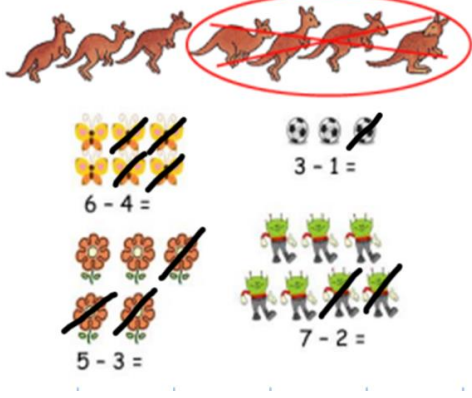
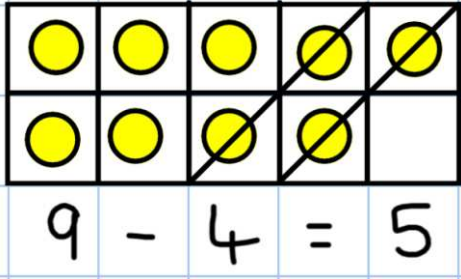
Year 6

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Add numbers with more than four-digits up to ten million using the compact column addition method with and without exchanging.</p>	<p>Children continue to use dienes or place value counters to add, exchange ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Children should have abstract supported by a pictorial or concrete if needed.</p> 
<p>Add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.</p>	<p>Introduce decimal place value counters and model exchange for addition.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Show in pictorial form, exchanging hundredths for tenths and tenths to 1s if necessary.</p>  <p>**The calculation must be shown above or alongside the manipulatives to see the connections between them.</p>	<p>Insert zeros for place holders when solving decimal problems where the addends do not have the same number of decimal places.</p> 

Vocabulary	addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point
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Subtraction

Early Years Foundation Stage

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Knows that a group of things change quantity when something is taken away.</p> <p>Find one less from a group of five objects, then ten objects.</p> <p>In practical activities and discussions, beginning to use the vocabulary involved in subtracting.</p> <p>Using quantities and objects, they subtract 2 single-digit numbers and count back to find the answer.</p>	<p>Use toys and general classroom resources for children to physically manipulate, group and regroup objects.</p>  <p>Use specific maths resources such as multilink cubes, numicon, bead strings etc.</p>  <p>Use visual supports such as ten frames, part part-whole models, subtraction mats with the physical objects and resources that can be manipulated.</p> 	<p>A group of pictures for children to cross out or cover quantities to support subtraction.</p>  <p>Use visual supports such as ten frames, part part-whole models and basic bar models with pictures/icons.</p>	<p>A focus on symbols and numbers to form a calculation.</p>  <p>**There is no expectation for children to be able to record a number sentence/subtraction sentence.</p>

Year 1

Objectives/Strategy

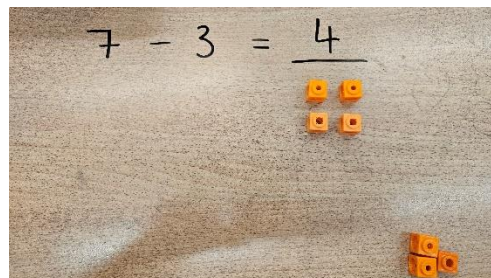
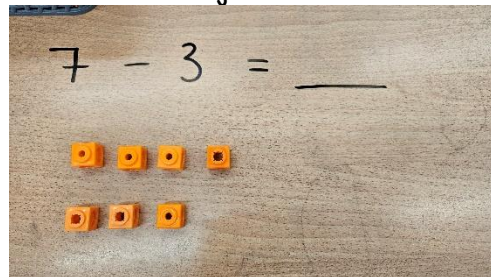
Concrete

Pictorial

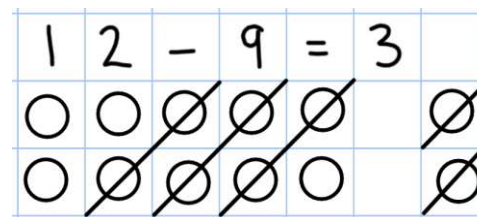
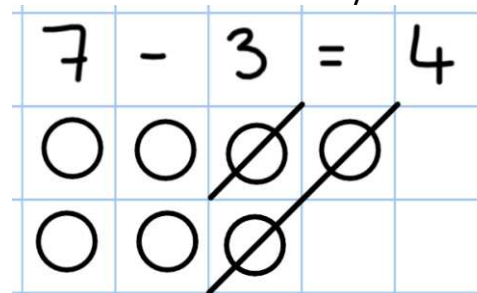
Abstract

Subtract one-digit and two-digit numbers to 20, including 0 by taking away ones.

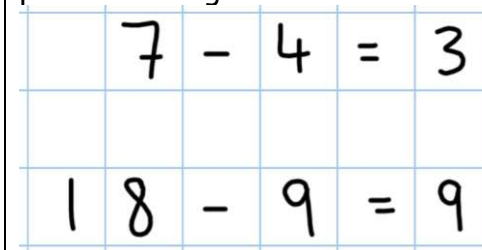
Use physical objects, counters, cubes etc to show how objects can be taken away.



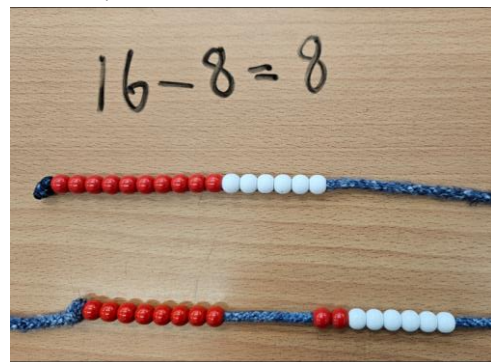
Cross out drawn objects to show what has been taken away.



Mentally solve subtraction problems using number sentences.

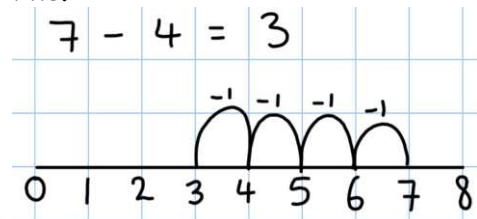


Make the larger number in the subtraction sentence. Move the beads along your bead string or rekenrek as you count backwards in ones.

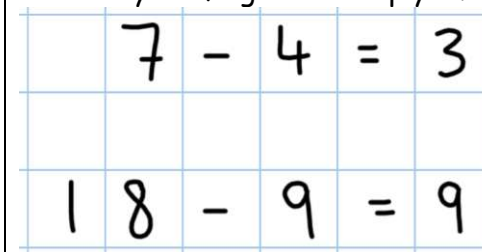


Counting back

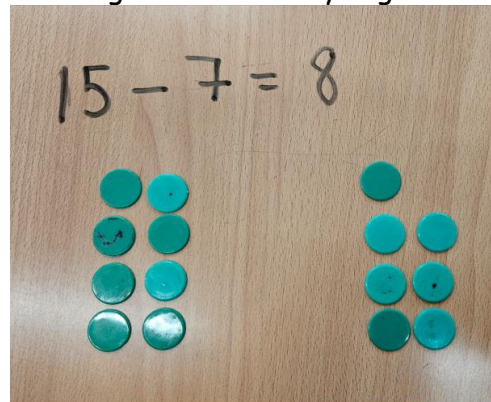
Count back on a number line or track. Start at the bigger number and count back the smaller number showing the jumps on the number line.



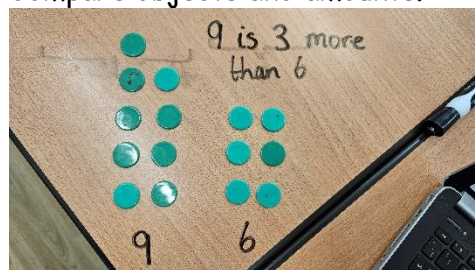
Put the starting number in your head. Then count back the subtrahend. What number are you at? Use your fingers to help you.



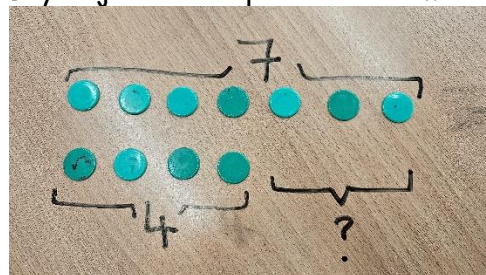
Use counters and move them away from the main group as you take them away counting backwards as you go.



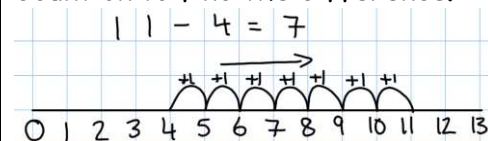
Compare objects and amounts.



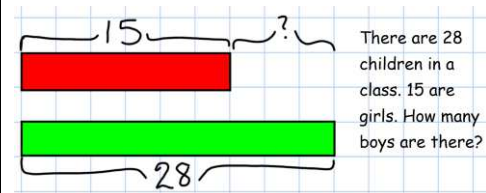
Lay objects to represent bar models.



Count on to find the difference.



Draw bars to find the difference between two numbers.



Put the smallest number in your head. Count on using your fingers until you get to the biggest number.

$$6 + 4 = 10$$

$$7 + 6 = 13$$

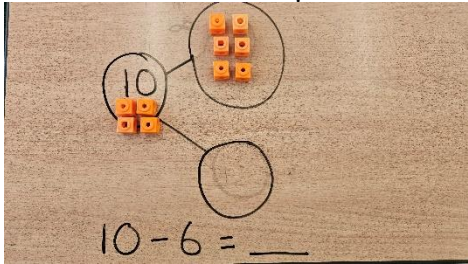
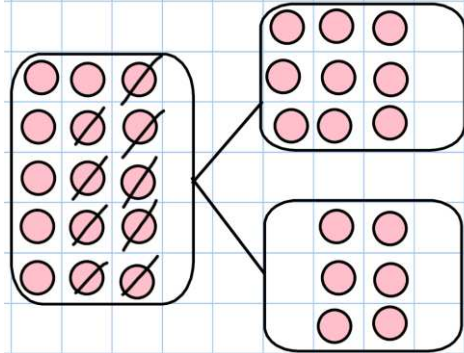
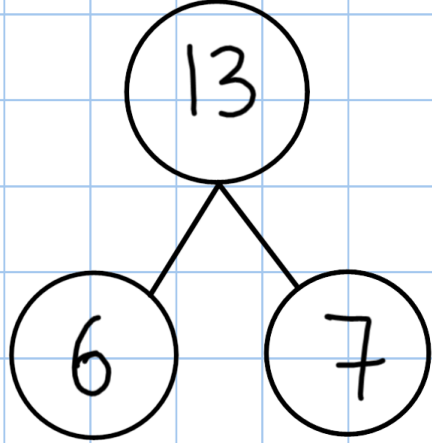

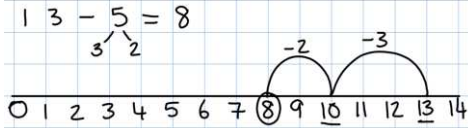
Find the difference

Represent and use number bonds and related subtraction facts within 20

Link to addition. Use the part part-whole models to model the inverse of addition.

Use a pictorial representation of objects to show the part part-whole model.

Move to using numbers within the part part-whole model.

<p>using part-part whole models.</p>	<p>If x is the whole and y is one of the parts, what must the other part be?</p> 	 <p>$15 - 9 = 6$</p>	
<p>Make 10</p>	<p>Use ten frames to solve subtraction problems where children jump back to 10 by removing all cubes within the frame. Then take away more until you reach the subtrahend.</p> <p>$14 - 5 = 9$</p> 	<p>Use a number line to jump back to the 10 (similarly to the ten frame). Then count back the remainder. Children will need to be able to partition the subtrahend into parts to jump back to the ten easily.</p> <p>$13 - 5 = 8$</p> 	<p>Children to solve make 10 problems mentally by jumping back to the 10 and then some more.</p> <p>$16 - 8 = 8$</p> <p>$16 - 6 = 10$</p> <p>$10 - 2 = 8$</p>
<p>Vocabulary</p>	<p>equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...</p>		

Year 2

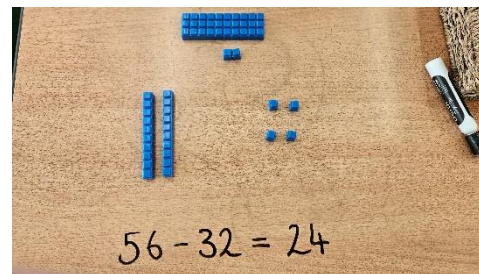
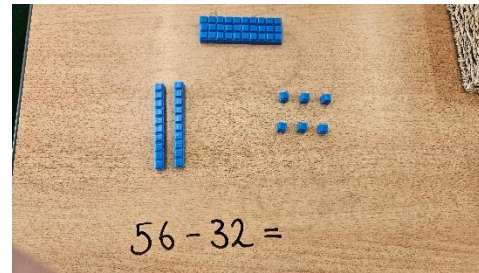
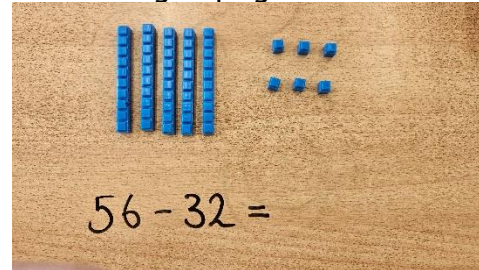
Objectives/Strategy

Concrete

Pictorial

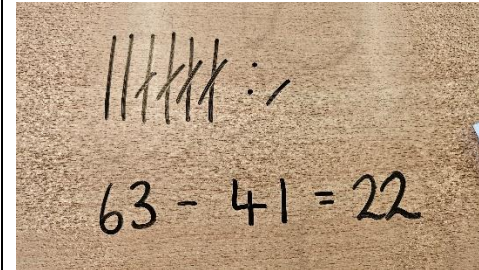
Abstract

Use dienes to show how to partition the number when subtracting without regrouping.



**The calculation must be shown above or alongside the manipulatives used.

Children to draw representations of dienes as sticks and dots and cross off.



Recording subtraction in columns support place value and prepares for formal written methods with larger numbers.

Children to use the more formal partition method to solve subtraction problems.

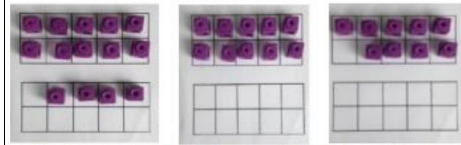
5	7	-	3	4	=	2	3
5	0	-	3	0	=	2	0
	7	-		4	=		3
2	0	+		3	=	2	3

Subtract a two-digit number and ones, a two-digit number and tens and 2 two-digit numbers using partitioning to subtract. (No regrouping)

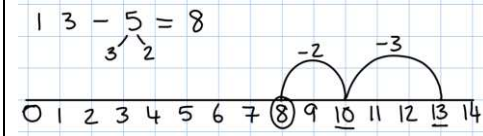
Make ten strategy

Use a bead string to model counting to the next ten and then the rest.

$$14 - 5 = 9$$



Use a number line to count on to the next ten and then the rest.



Solve make ten questions mentally using the strategy of making the next ten and adding more.

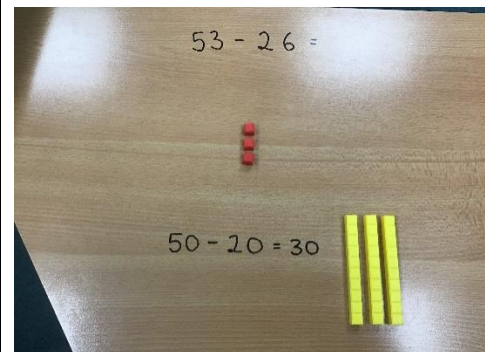
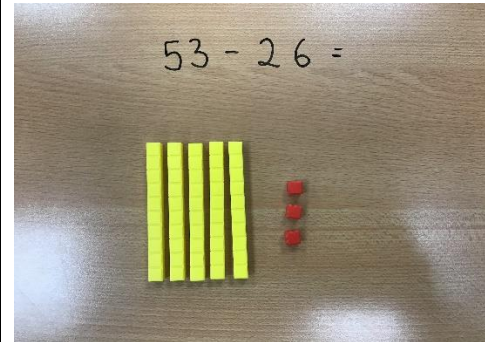
$$16 - 8 = 8$$

$$16 - 6 = 10$$

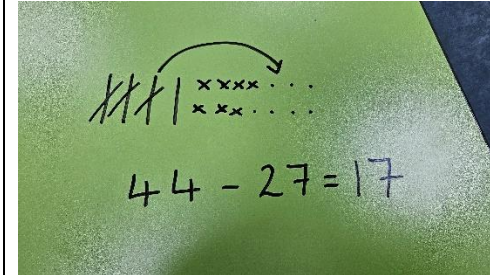
$$10 - 2 = 8$$

Subtract a two-digit number and ones, a two-digit number and tens and 2 two-digit numbers using partitioning to subtract. (with regrouping)

Use dienes to show how to partition the number when subtracting with regrouping. Model using the phrase 'take and make'



Children to draw representations of dienes as sticks and dots and cross off.



Recording subtraction in columns support place value and prepares for formal written methods with larger numbers.

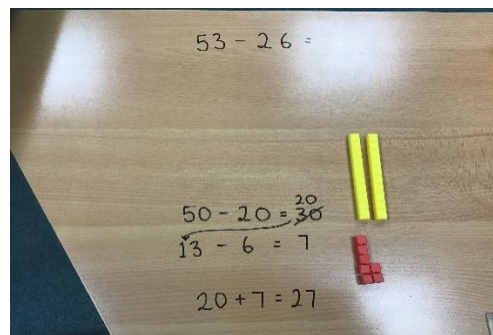
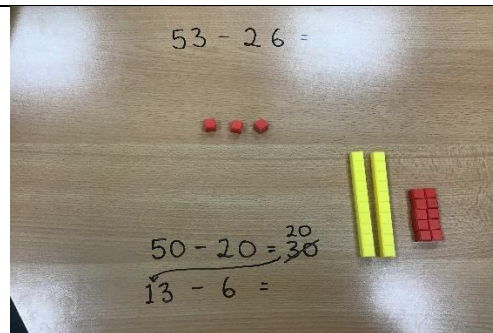
Children to use the more formal partition method to solve subtraction problems - including the concept of exchanging.

$$72 - 36 = 36$$

$$70 - 30 = 40$$

$$12 - 6 = 6$$

$$30 + 6 = 36$$

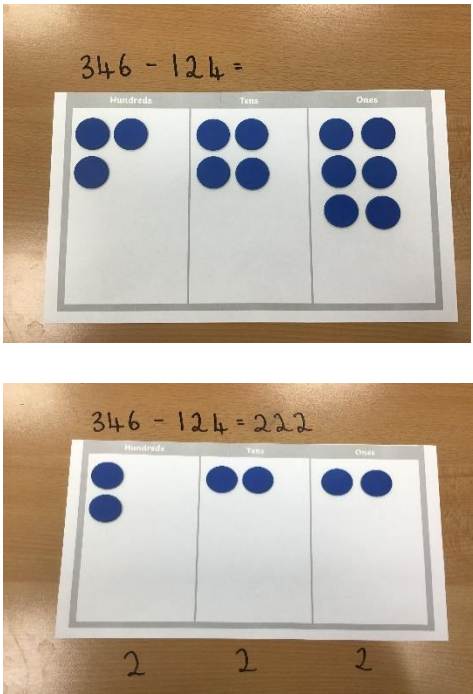
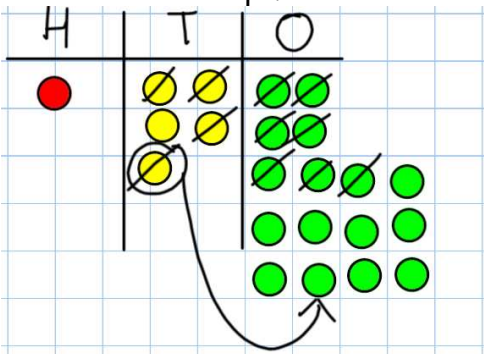
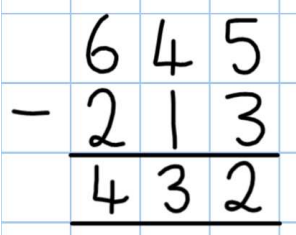


**The calculation must be shown above or alongside the manipulatives used.

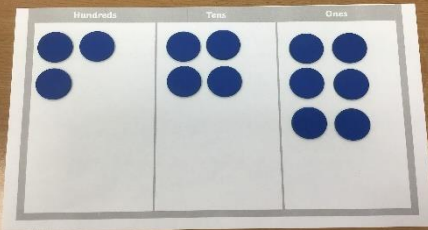
Vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

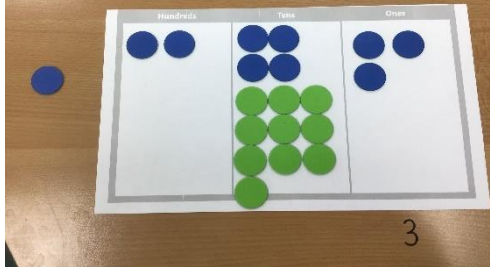
Year 3

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>To subtract numbers with up to three-digits, using column subtraction (without exchanging).</p>	<p>Use dienes to model.</p>  <p>** The calculation must be shown above or alongside the manipulatives used.</p>	<p>Children need to be secure with using place value counters before moving on to abstract concept.</p>  <p>156 - 37 = 119</p>	<p>Children to start by using the partition method used in Year 2.</p> $72 - 36 = 36$ $70 - 30 = 40$ $12 - 6 = 6$ $30 + 6 = 36$ <p>Once secure, children to move on to expanded column subtraction,</p> 
<p>To subtract numbers with up to three-digits, using column subtraction (with exchanging).</p>	<p>Begin with using dienes (see Year 2 subtraction concrete images). Move to then using place value counters, modelling exchanging of a ten into ten ones. Continue to model using the phrase 'take and make.'</p>	<p>When confident, children can attempt their own way to record pictorially (if needed). Most will move straight to abstract once the exchanging (borrowing, take and make) process is understood.</p>	<p>Children should begin with the column subtraction learned within Year 3. Make the link between the concept of exchanging in the partition method form Year 2 and the exchanging in the formal written method.</p>

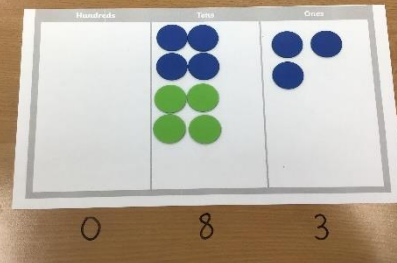
$$346 - 263 =$$



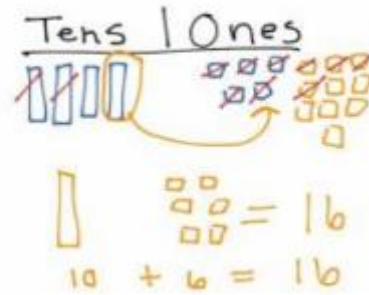
$$346 - 263 =$$



$$346 - 263 =$$



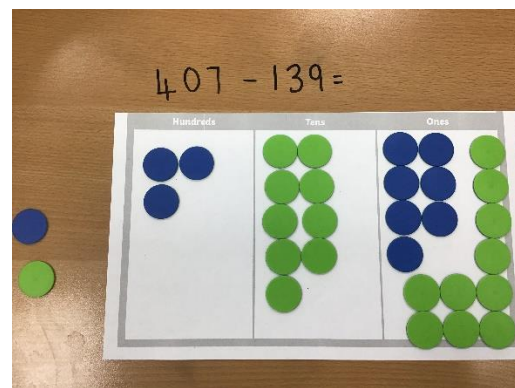
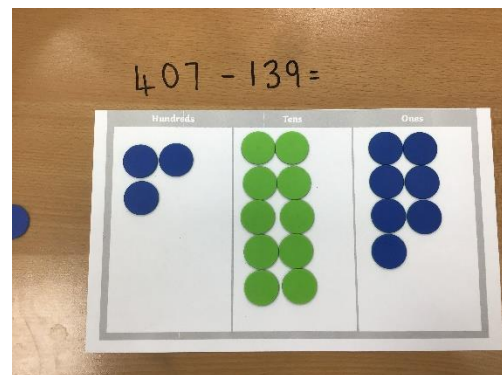
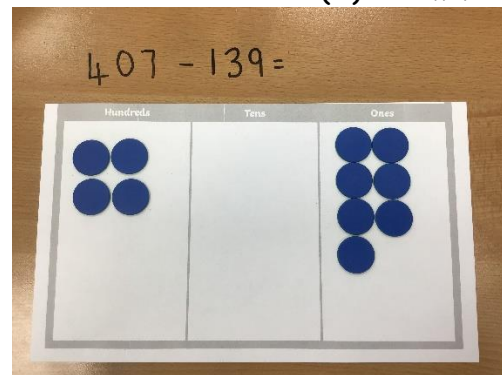
$$\begin{array}{r} 45 \\ -29 \\ \hline 16 \end{array}$$



	5	2	4
-	2	1	7
	<u>3</u>	<u>0</u>	<u>7</u>

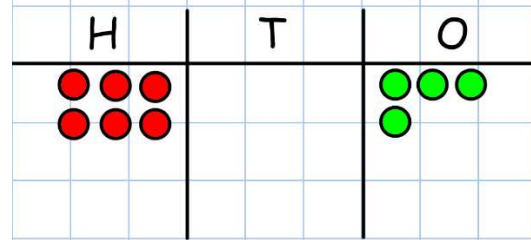
To subtract numbers with up to three-digits using column subtraction where they need to exchange from 2 columns away. (Over the 0)

Use counters to model exchanging over the 0 in the tens (T) column.

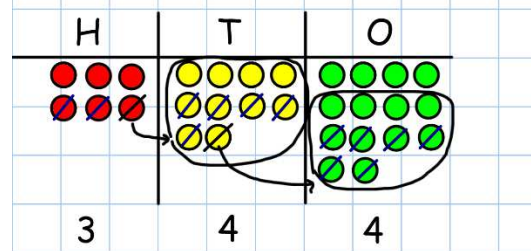


Children need to be secure with using place value counters before moving on to abstract concept.

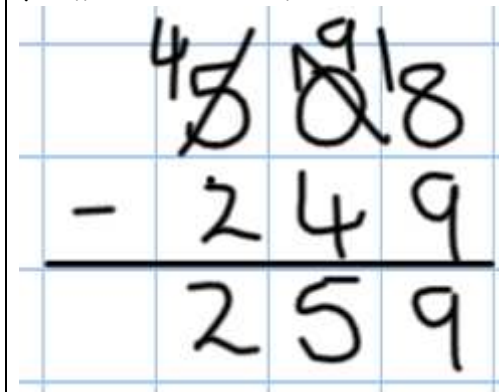
$$604 - 256 =$$

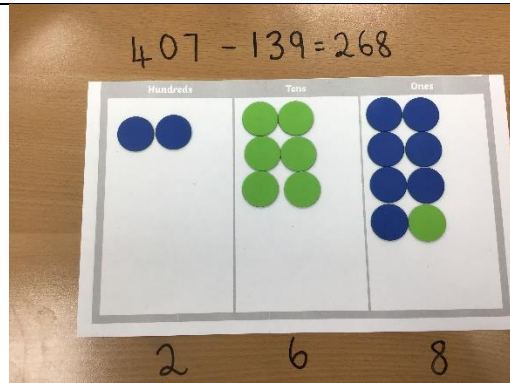


$$604 - 256 = 344$$



Children to use column subtraction method with exchanging but going across the Tens column to borrow from the Hundreds.





Vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

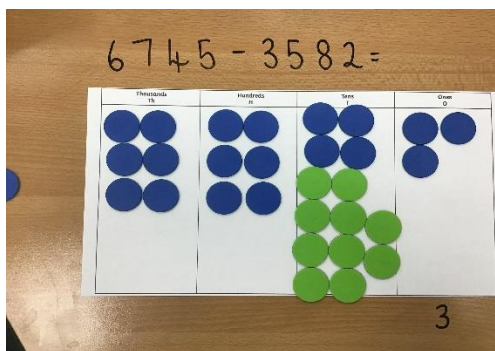
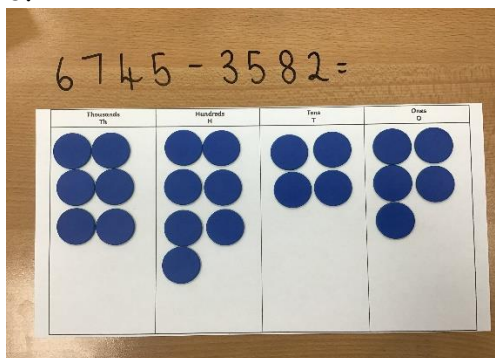
Year 4

Objectives/Strategy

Subtract numbers with up to 4 digits using the formal compact column subtraction method.

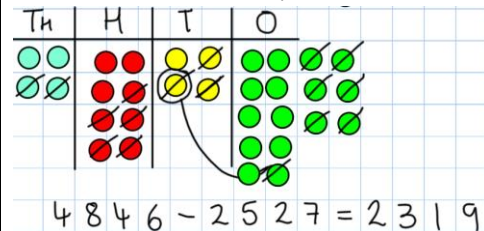
Concrete

Model process of exchange using dienes (see Year 3 if necessary) and then move on to place value counters. Continue to use the phrase 'take and make' for exchange - see Years 2 and 3.



Pictorial

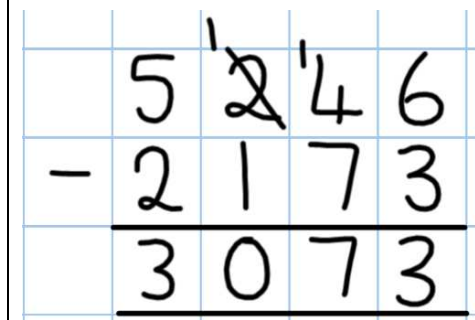
Children to draw place value counters and show their exchange - see Years 2 and 3.



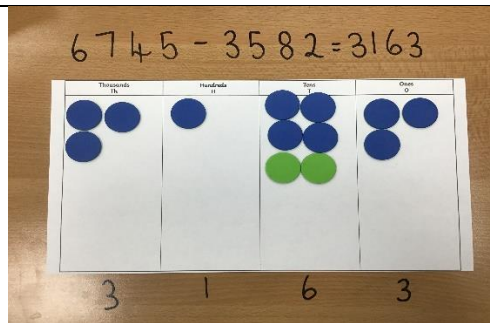
**The calculation must be shown above or alongside the manipulatives used.

Abstract

The children will build on their understanding of column subtraction (from Year 3) to compact column subtraction.

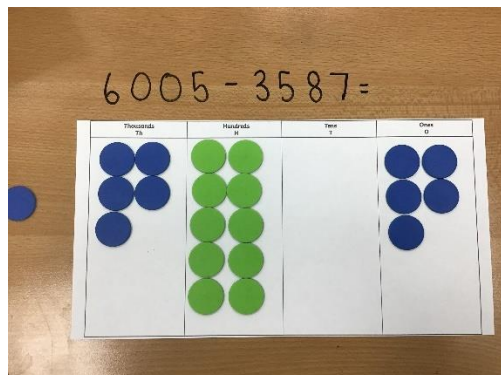
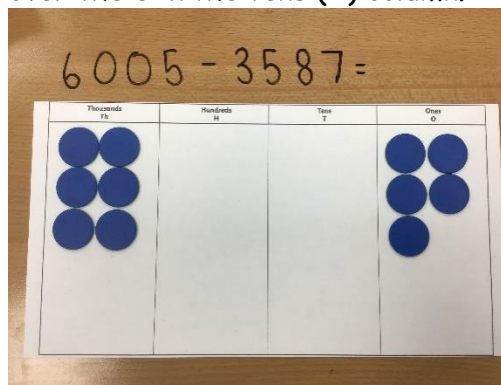


This will lead to an understanding of subtracting any numbers, regardless of size, including decimal numbers.



** The calculation must be shown above or alongside the manipulatives used.

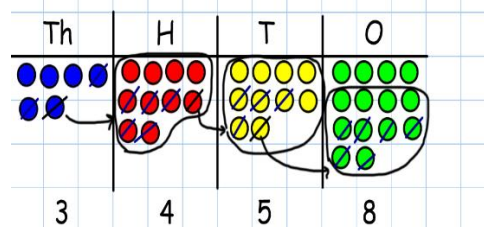
Use counters to model exchanging over the 0 in the tens (T) column.



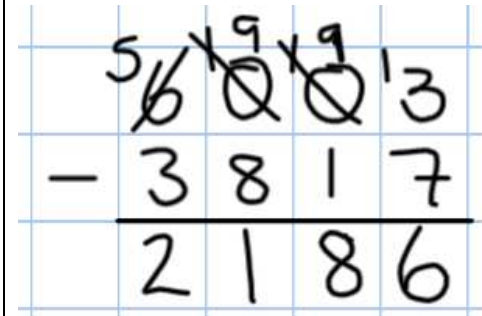
To subtract numbers with up to three-digits using column subtraction where they need to exchange from 2 or 3 columns away. (Over the 0)

Children need to be secure with using place value counters before moving on to abstract concept.

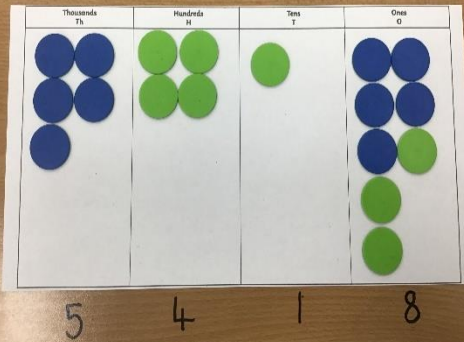
$6004 - 2546 = 3458$



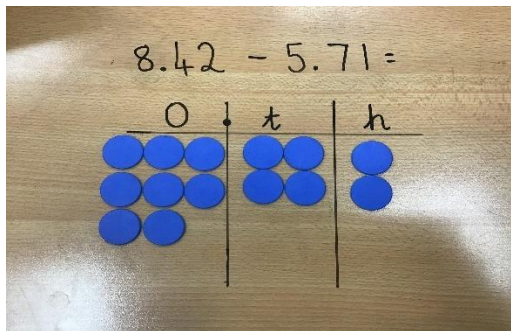
Children to use column subtraction method with exchanging but going across the Tens column to borrow from the Hundreds.



$$6005 - 3587 = 5418$$

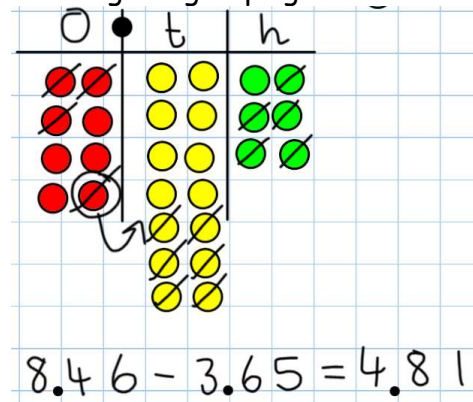


Children to be encouraged to use place value counters to represent numbers and take counters away to subtract.

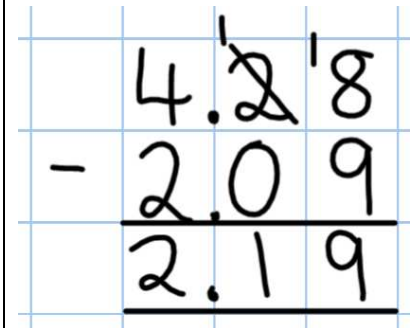


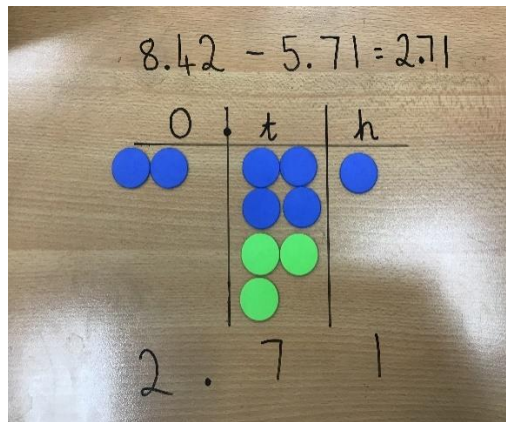
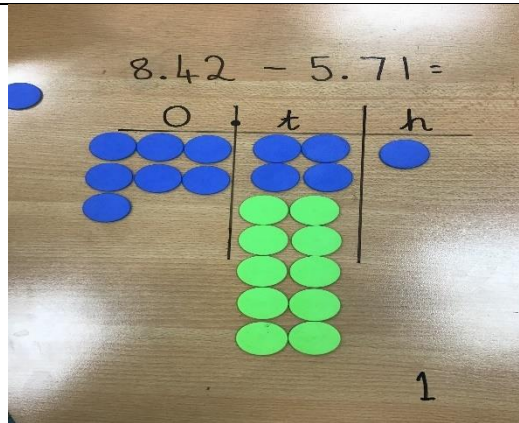
Introduce decimal subtraction through the context of money.

When confident, children can find their own way to record the exchange/regrouping.



Using the column subtraction method will lead to an understanding of subtracting decimal numbers.





Vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Year 5

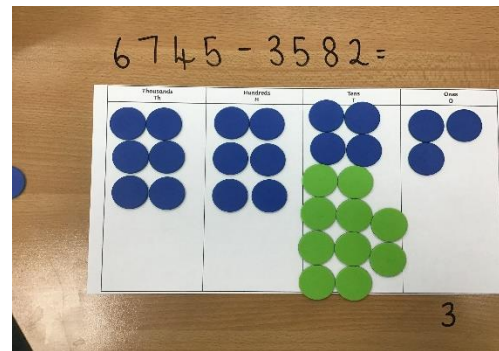
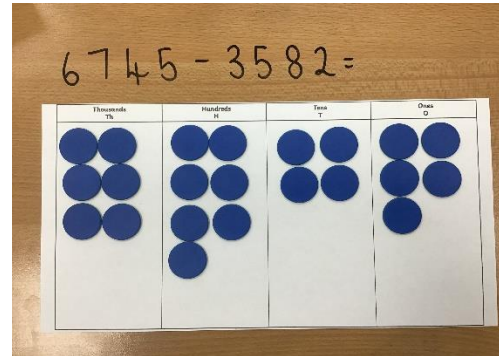
Objectives/Strategy

Subtract with at least four-digit numbers, including money and measures.

Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal places).

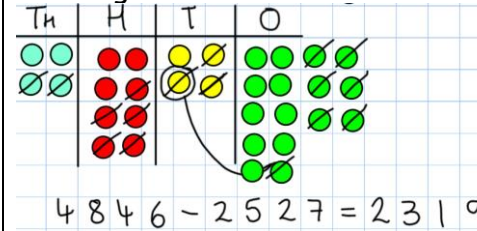
Concrete

If necessary, model the process of exchange using numicon or dienes and then move on to place value counters. Continue to use the phrase 'take and make' for exchange - see Years 2 and 3.



Pictorial

If necessary, children to draw place value counters and show their exchange - see Years 2 and 3.



**The calculation must be shown above or alongside the manipulatives used.

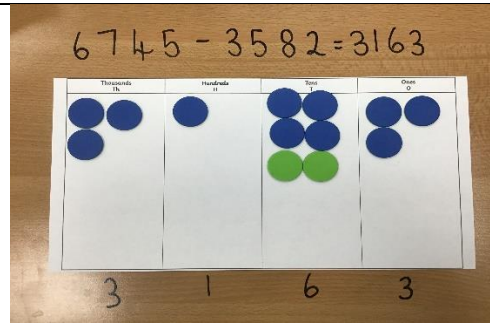
Abstract

The children will build on their understanding of expanded column subtraction (from Year 3) to column subtraction.

$$\begin{array}{r} 23187421 \\ - 194354 \\ \hline 193067 \end{array}$$

This will lead to an understanding of subtracting any numbers, regardless of size, including decimal numbers.

$$\begin{array}{r} 4521.538 \\ - 379.643 \\ \hline 4141.895 \end{array}$$



** The calculation must be shown above or alongside the manipulatives used.

Vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Year 6

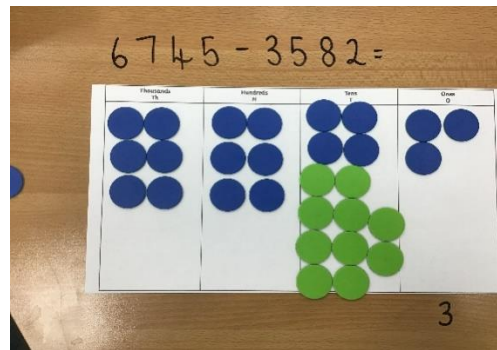
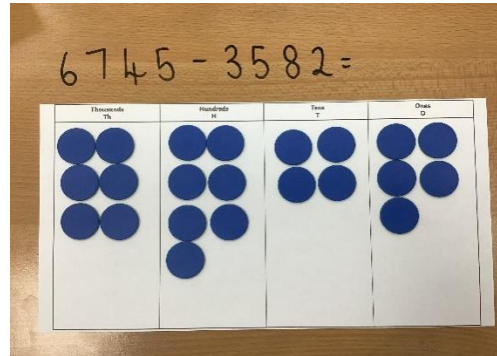
Objectives/Strategy

Subtract with at least four-digit numbers, including money and measures.

Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal places).

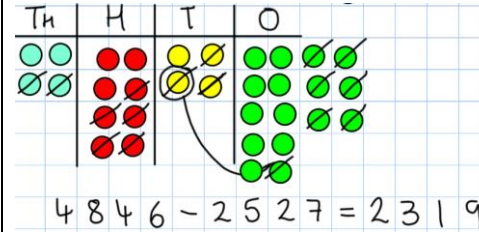
Concrete

If necessary, model process of exchange using numicon or dienes and then move on to place value counters. Continue to use the phrase 'take and make' for exchange - see Years 2 and 3.



Pictorial

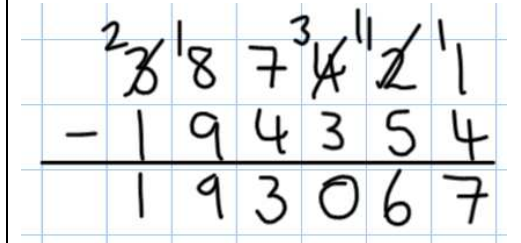
If necessary, children to draw place value counters and show their exchange - see Years 2 and 3.



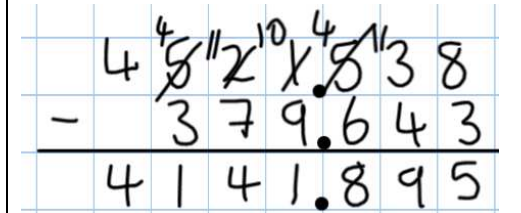
**The calculation must be shown above or alongside the manipulatives used.

Abstract

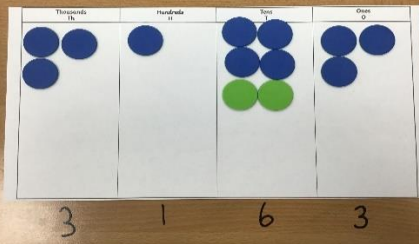
The children will build on their understanding of expanded column subtraction (from Year 3) to column subtraction.



This will lead to an understanding of subtracting any numbers, regardless of size, including decimal numbers.



$$6745 - 3582 = 3163$$




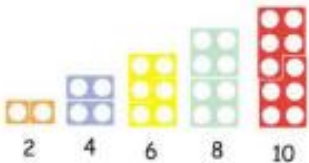


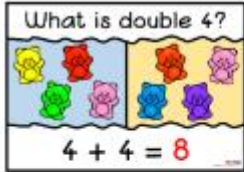

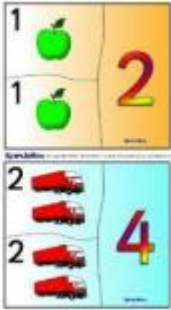
** The calculation must be shown above or alongside the manipulatives used.

Vocabulary

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Multiplication

Early Years Foundation Stage

Objectives/Strategy	Concrete	Pictorial	Abstract															
<p>Solve problems including doubling</p>	<p>Counting and other maths resources for children to make 2 equal groups.</p>  <p>Physical and real life examples that encourage children to see concept of doubling as adding two equal groups.</p>   	<p>Pictures and icons that encourage children to see concept of doubling as adding two equal groups.</p>   	<p>Addition calculations to model adding two equal groups.</p> <table border="1" style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td style="width: 33%;">1</td> <td style="width: 33%;">+</td> <td style="width: 33%;">1</td> </tr> <tr> <td>2</td> <td>+</td> <td>2</td> </tr> <tr> <td>3</td> <td>+</td> <td>3</td> </tr> <tr> <td>4</td> <td>+</td> <td>4</td> </tr> <tr> <td>5</td> <td>+</td> <td>5</td> </tr> </table>	1	+	1	2	+	2	3	+	3	4	+	4	5	+	5
1	+	1																
2	+	2																
3	+	3																
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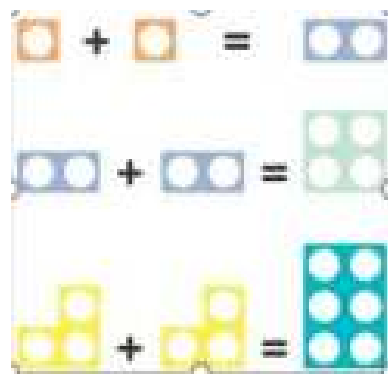
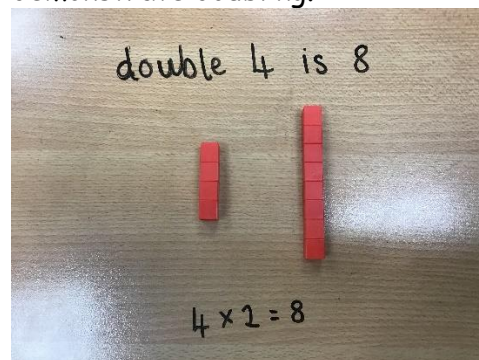
Year 1

Objectives/Strategy

Doubling

Concrete

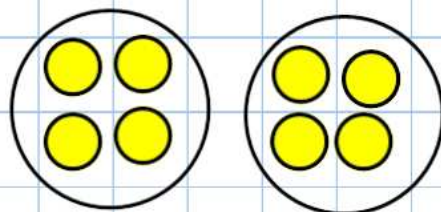
Use practical activities using manipulatives including multilink cubes, counters and numicon to demonstrate doubling.



Pictorial

Draw pictures to show how to double numbers.

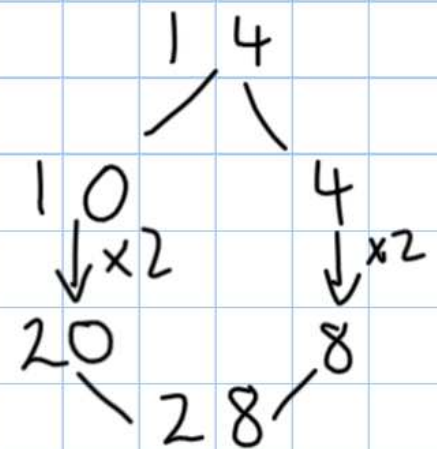
Double 4 is 8

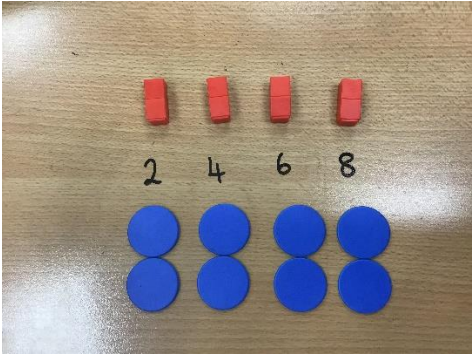
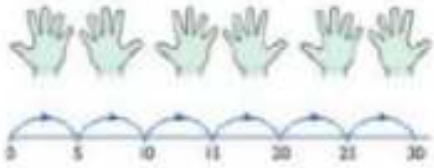





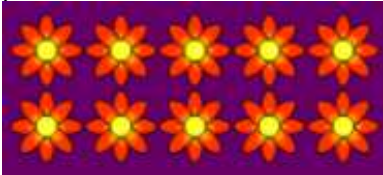
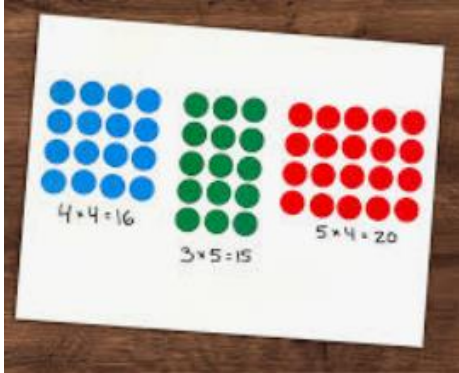
Abstract

Partition a number and then double each part before recombining it back together.

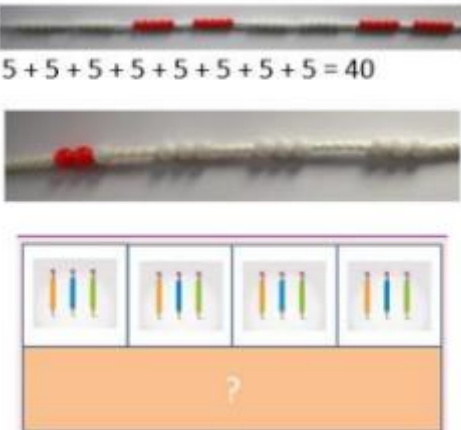
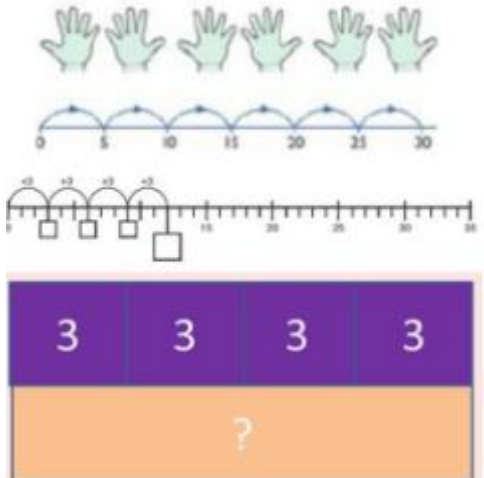

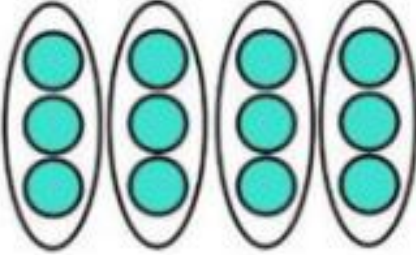

Double 14 is 28



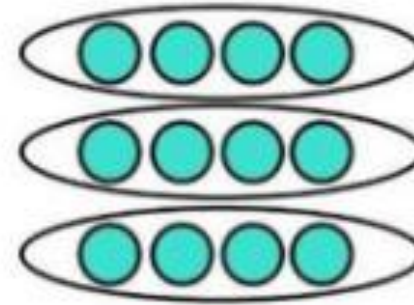
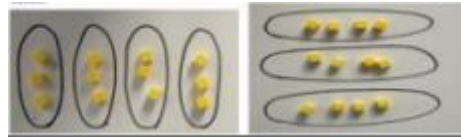
<p>Counting in multiples</p>	<p>Count in multiples supported by using concrete objects in equal groups such as multilink cubes, dienes, counters etc.</p> 	<p>Use a number line or pictures to continue supporting counting in multiples.</p> 	<p>Count in multiples of a number aloud and then, write sequences with multiples of numbers.</p> <p style="text-align: center;">2, 4, 6, 8, 10</p> <p style="text-align: center;">5, 10, 15, 20, 25, 30</p>
<p>Repeated addition</p>	<p>Use different objects to add equal groups.</p> 	<p>Use a number line to show repeated addition.</p> 	<p>Write addition sentences to describe objects and pictures.</p> 

<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to multiplication problems.</p> 	<p>Use dots (as pictorial representation) to understand arrays.</p> 	<p>Write down arrays as multiplication problems using the multiplication symbol.</p> $3 \times 2 = 6$ $2 \times 5 = 10$
<p>Vocabulary</p>	<p>Groups of, lots of, times, array, altogether, multiply</p>		

Year 2

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Counting in multiples of 2, 3, 5 and 10 from 0 using repeated addition.</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models to support concrete objects.</p>  <p>$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p>	<p>Children to use pictorial representations such as number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p>$4 \times 3 = \square$</p>
<p>Understand multiplication is commutative</p>	<p>Create arrays using counters, multilink cubes and numicon.</p>  <p>Pupils should remember from Year 1 that an array can be represented in different equations and that, as multiplication is commutative, the</p>	<p>Use representation of arrays to show different calculations and explore commutativity.</p> 	<p>Use an array to write down multiplication sentences.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>

order of the multiplication does not affect the answer.

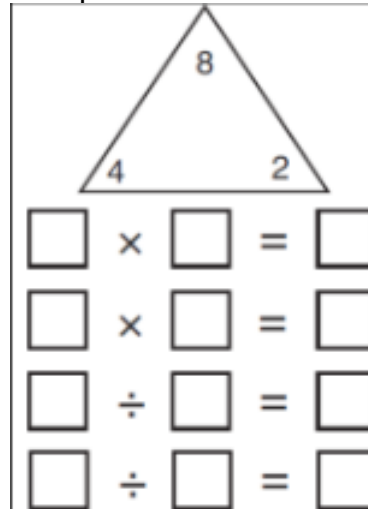


Using the inverse. (This is taught alongside division so pupils understand how multiplication and division are related).

Use concrete objects to show grouping as the opposite to multiplication.



Use fact family pictorials to show the relationship between multiplication and division.



Show all 8 related multiplication and division facts for a given pictorial representation in multiplication and division sentences.

$2 \times 4 = 8$

$4 \times 2 = 8$

$8 \div 2 = 4$

$8 \div 4 = 2$

$8 = 2 \times 4$

$8 = 4 \times 2$

$2 = 8 \div 4$

$4 = 8 \div 2$

Show all 8 related fact family sentences.

Vocabulary

Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.

Year 3

Objectives/Strategy

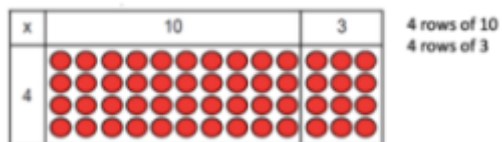
Concrete

Pictorial

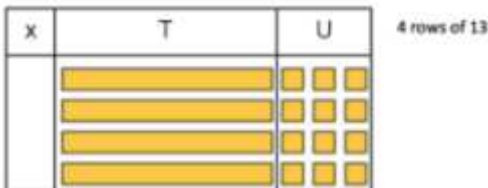
Abstract

Multiplying a two-digit number by a one-digit number using the expanded column multiplication method.

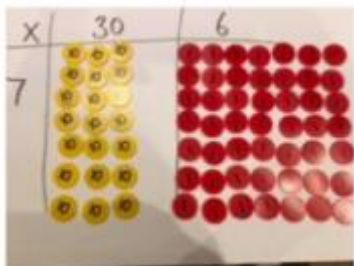
Show the link to arrays to first introduce the grid method. (see below).



Move on to using dienes to move towards a more compact method.

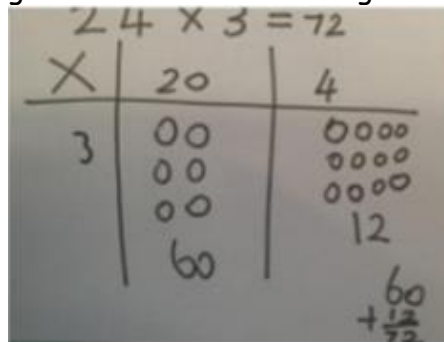


Then move on to using place value counters within the grid.

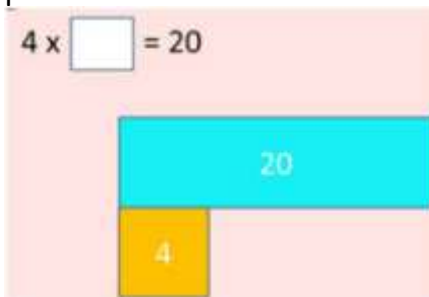


Add up each column starting with the ones making any exchanges as needed.

Children can represent their work with place value counters in a way that they understand. The children can draw the counters using colours to show the different amounts of just use the circles in different parts of the grid to show their thinking.

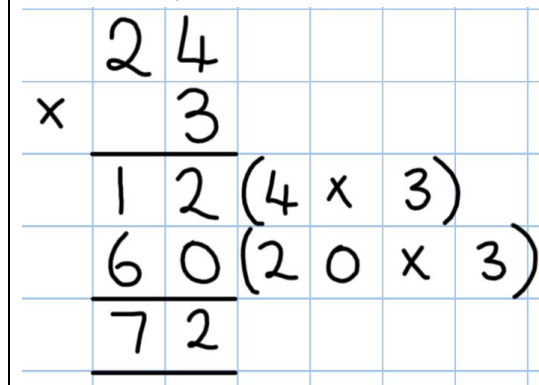


Children to use bar models to help them explore missing number problems.



Start with multiplying by one-digit numbers and showing the clear addition alongside the grid.

Children to add up each column to find the answer.



	**The calculation must be shown above or alongside the manipulatives used.		
Vocabulary	Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up		

Year 4

Objectives/Strategy

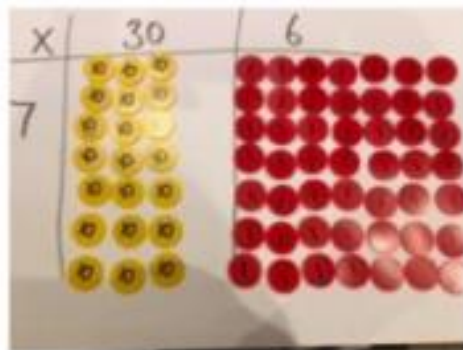
Concrete

Pictorial

Abstract

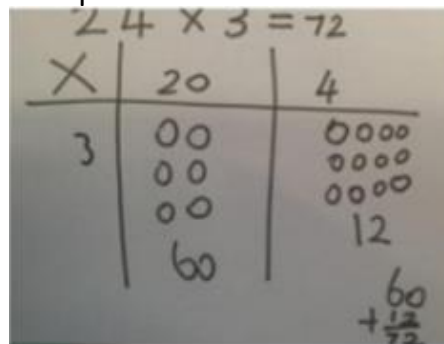
Multiply two-digit and three-digit numbers by a one-digit number using the compact column multiplication method.

Children can continue to be supported by place value counters at the stage of multiplication. This initially can be done where there is no regrouping.

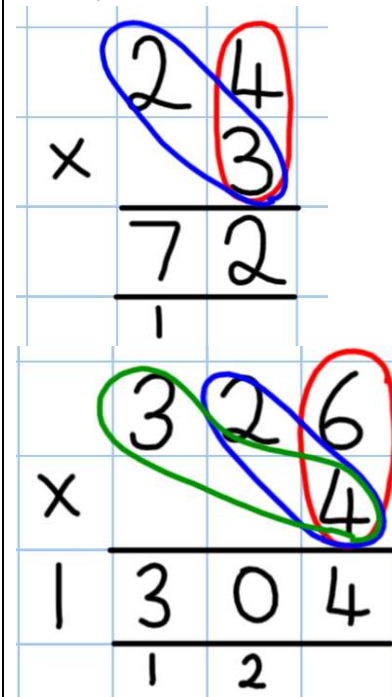


**The calculation must be shown above or alongside the manipulatives used.

Use the grid method (learned in Year 3) as a bridge to the new expanded column method for multiplication.



Children to use the compact column multiplication method as outlined below.



Vocabulary

Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive

Year 5

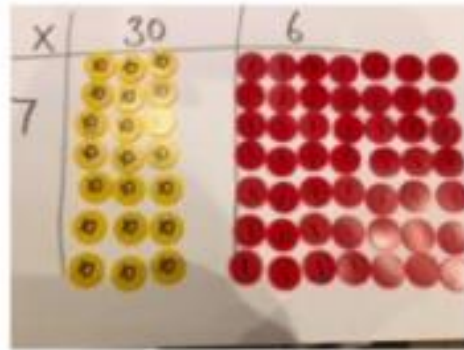
Objectives/Strategy

Concrete

Pictorial

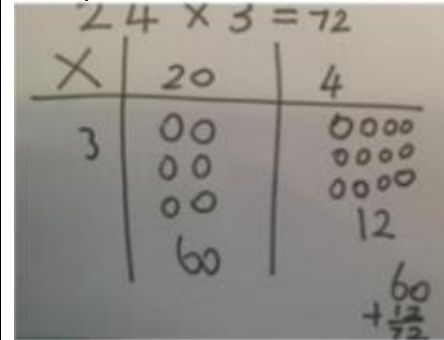
Abstract

Children can continue to be supported by place value counters at the stage of multiplication. This initially can be done where there is no regrouping.

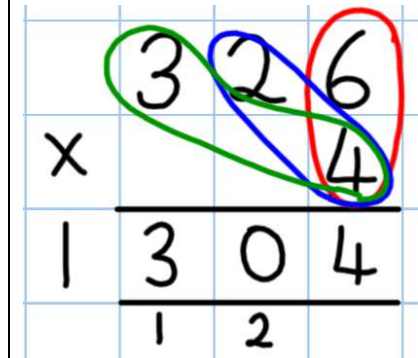
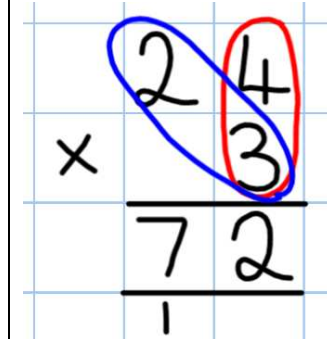


**The calculation must be shown above or alongside the manipulatives used.

Use the grid method (learned in Year 3) as a bridge and the expanded column multiplication method (learned in Year 4) to the new compact column method for multiplication.



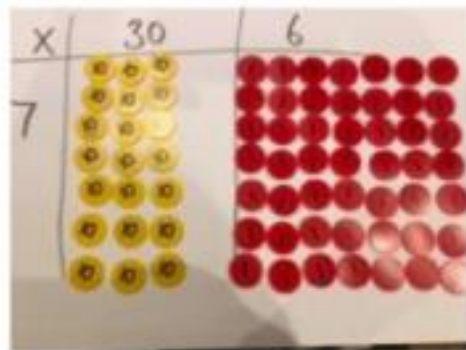
Children to be introduced to the compact column multiplication method as outlined below.



Multiply numbers up to four-digits by a one-digit number using the column multiplication method.

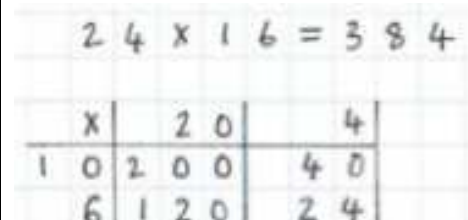
Multiply numbers up to four-digits by a two-digit number using the column multiplication (long) method.

Children can continue to be supported by place value counters at the stage of multiplication. This initially can be done where there is no regrouping.

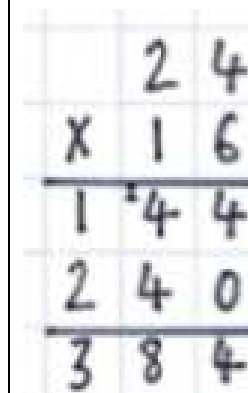


**The calculation must be shown above or alongside the manipulatives used.

For 2-digit multipliers, start by showing the different rows which would have been needed in the grid method learned in Year 3.



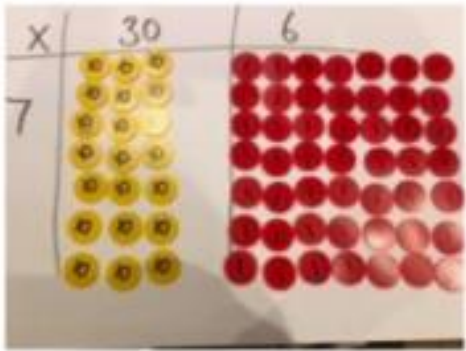
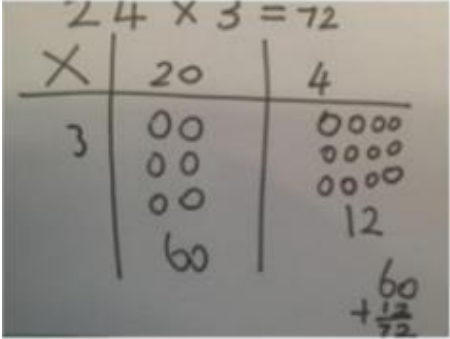
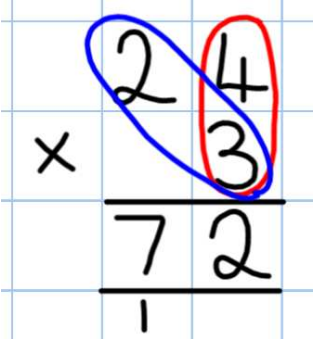
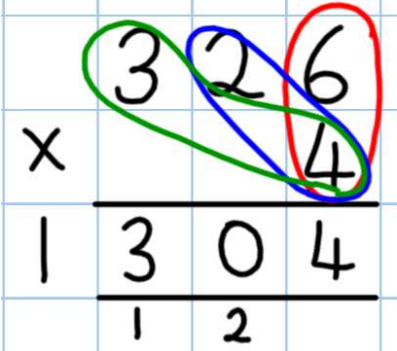
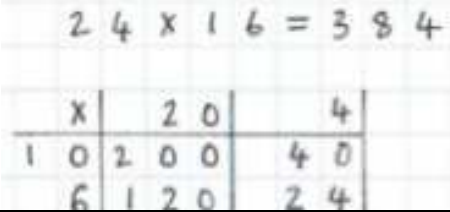
Children to be then introduce the second row within the compact column multiplication methods as outlined below. Make the link between the need for the second row with the grid method in the pictorial representation.

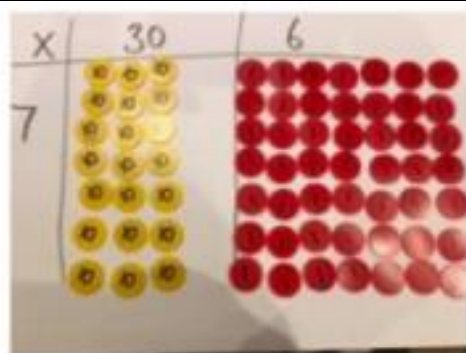


Vocabulary

Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

Year 6

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Multiply numbers up to four-digits by a one-digit number using the column multiplication method.</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially can be done where there is no regrouping.</p>  <p>**The calculation must be shown above or alongside the manipulatives used.</p>	<p>Use the grid method (learned in Year 3) as a bridge and the expanded column multiplication method (learned in Year 4) to the new compact column method for multiplication.</p> 	<p>Children to be introduced to the compact column multiplication method as outlined below.</p>  
<p>Multiply numbers up to four-digits by a two-digit number using the column multiplication (long) method.</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially can be done where there is no regrouping.</p>	<p>For 2-digit multipliers, start by showing the different rows which would have been needed in the grid method learned in Year 3.</p> 	<p>Children to be then introduce the second row within the compact column multiplication methods as outlined below. Make the link between the need for the second row with the grid method in the pictorial representation.</p>



**The calculation must be shown above or alongside the manipulatives used.

$$\begin{array}{r}
 24 \\
 \times 16 \\
 \hline
 144 \\
 240 \\
 \hline
 384
 \end{array}$$

Multiply decimals up to 2 decimal places by a single digit.

Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer.




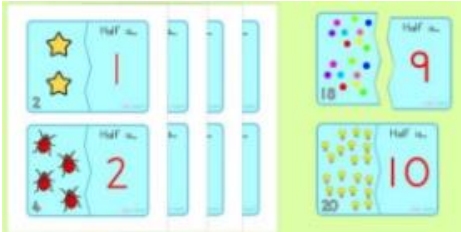
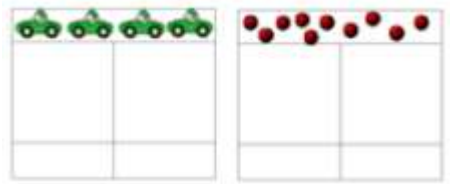
$$\begin{array}{r}
 3.19 \\
 \times 8 \\
 \hline
 25.52
 \end{array}$$

Vocabulary

Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed

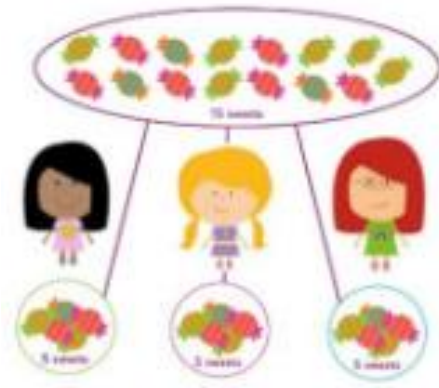
Division

Early Years Foundation Stage

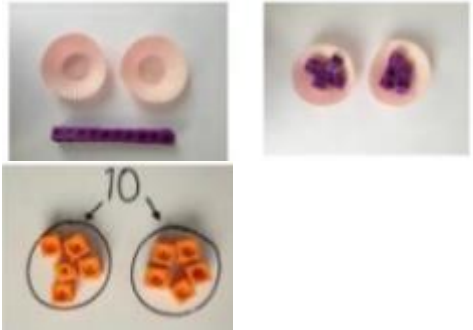
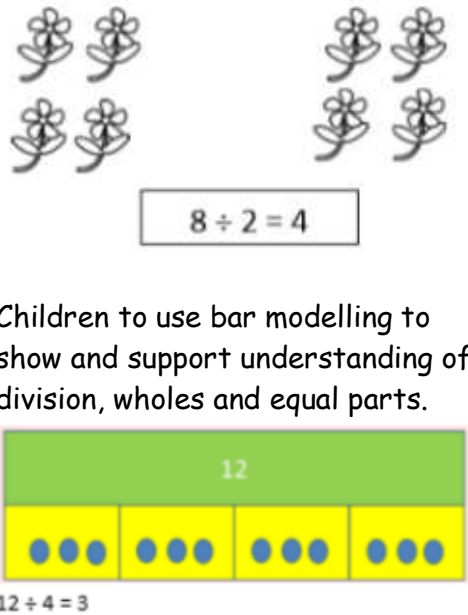
Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Solve problems including halving and sharing.</p> <p>Halving should include:</p> <ul style="list-style-type: none"> - Halving a whole - Halving a quantity of objects. <p>Sharing a quantity of objects.</p>	<p>Children have the opportunities to physically cut objects, foods or shapes in half.</p>   <p>Counting and other maths resources for children to share into two equal groups.</p>  <p>Use visual supports such as halving mats and part part-whole models with manipulatives (physical objects).</p>	<p>Pictures and icons that encourage children to see the concept of halving in relation to subitising, addition and subtraction knowledge. An example of this would be knowing 6 is made of 3 and 3 so half of 6 is 3.</p>  <p>Using part part-whole models with pictures of icons to support understanding of finding 2 equal parts of a number to further understand how two halves make a whole.</p>  <p>Pictures for children to create and visualise 3 or more equal groups.</p>	







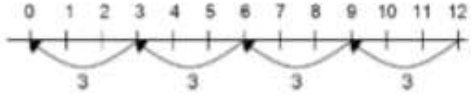
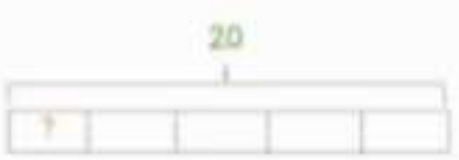
Counting and other maths resources for children to explore sharing with more than 2 equal groups.





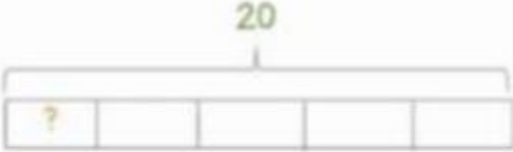

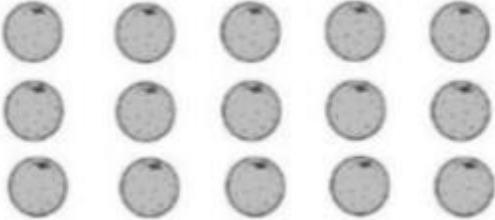
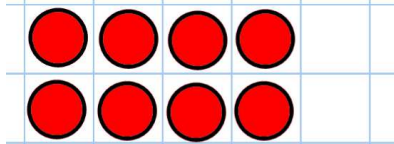
Year 1

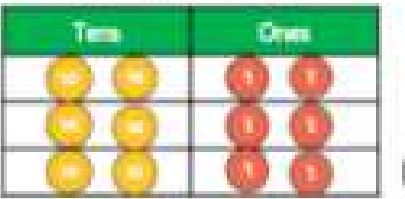
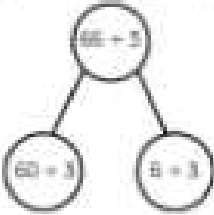
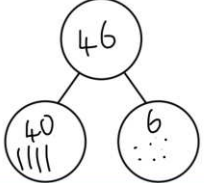
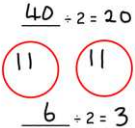
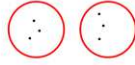

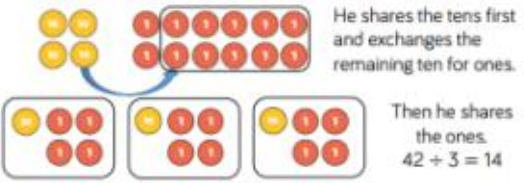
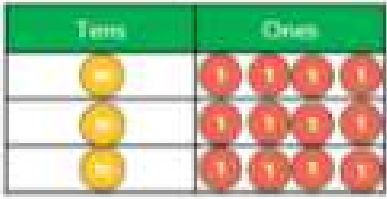
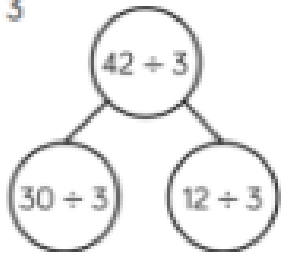
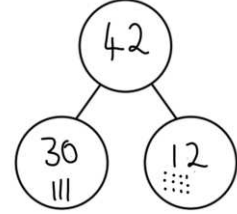


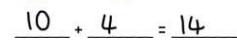
Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing (sharing objects into groups).</p>	<p>Using objects to share them into 2 equal groups.</p> 	<p>Children to use pictures or shapes to share amounts/ quantities.</p>  <p>Children to use bar modelling to show and support understanding of division, wholes and equal parts.</p>	<p>Share an amount between 2 or 3 equal groups and solve mentally with numbers up to 20 and construct number sentences to show this.</p> <p>Share 9 buns between three people.</p> $9 \div 3 = 3$
<p>Vocabulary</p>	<p>share, share equally, one each, two each..., group, groups of, lots of, array</p>		

Year 2

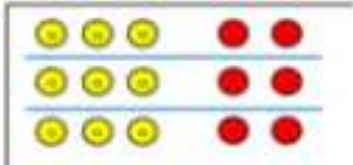
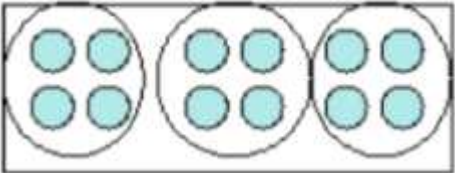
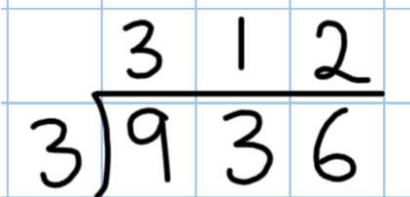
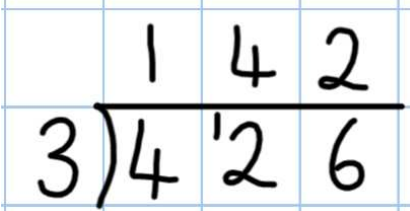
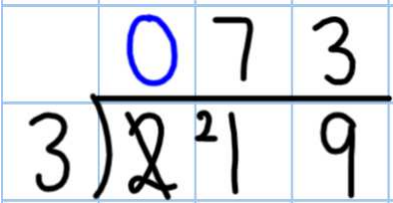
Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Divide quantities into equal groups.</p>  <p>Use multilink cubes, counters or objects to aid understanding of division into equal groups.</p>   	<p>Use a number line to show jumping back groups of a number. The number of jumps equals the number of groups.</p>  <p>Continue to use bar models to represent division. Think of the bar as a whole. Split the second bar into the number of equal groups required and then work out how many would be needed in each group. Link the bar model to the multiplication sentence too.</p>  <p> $20 \div 5 = ?$ $5 \times ? = 20$ </p>	<p>Use multiplication facts know about the 2s, 3s, 5s and 10s to solve division questions mentally.</p> <p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>
<p>Vocabulary</p>	<p>share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over</p>		

Year 3

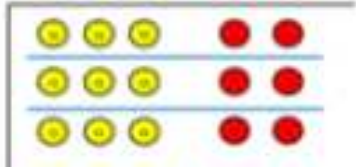
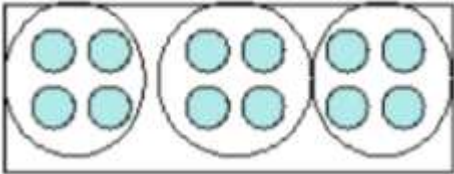
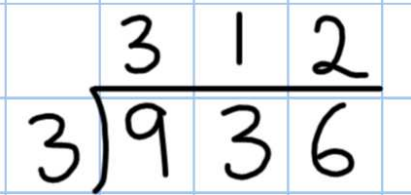
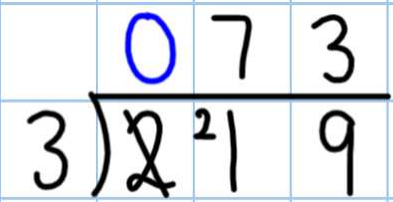
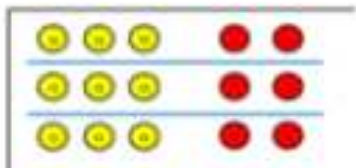
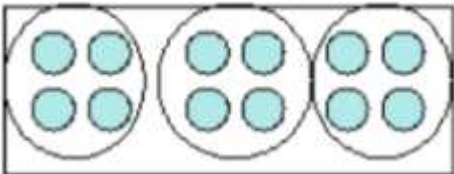
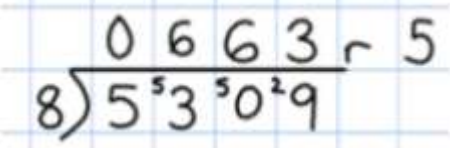
Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use multilink cubes, counters, objects or place value counters to aid understanding.</p>  $96 \div 3 = 32$ 	<p>Continue to use the bar model to aid solving division problems linking to known multiplication facts.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>Use multiplication facts know about the 2s, 3s, 4s, 5s, 8s and 10s to solve division questions mentally.</p> <p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	<p>Link division to multiplication arrays by creating arrays and thinking about the number sentences which can be created.</p>  <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$</p> <p> $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw arrays and use lines to split the arrays into groups to make multiplication and division sentences.</p> 	<p>Find the inverse of multiplication and division sentences by creating the eight related number sentences for a give array.</p>  $2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$

<p>Divide two-digit numbers by a one-digit number by partitioning tens and ones.</p>	<p>Use place value grids to solve division problems.</p> 	<p>Use part part-whole models to show division statements.</p> 	<p> $46 \div 2 =$  </p> <p> $\frac{40}{6} \div 2 = 20$  </p> <p> $\frac{6}{2} \div 2 = 3$  </p> <p> $20 + 3 = 23$  </p>
<p>Divide numbers that involve exchanging between the tens and ones with no remainders.</p>	<p>Use place value counters to divide 2 digit numbers into equal groups.</p> 	<p>Children to use pictorial representations for the place value counters alongside the part-whole models.</p>  <p>Children to use their times-tables knowledge to partition the number into multiples of the divisor.</p> 	<p> $42 \div 3 = 14$  </p> <p> $\frac{30}{12} \div 3 = 10$  </p> <p> $\frac{12}{2} \div 3 = 4$  </p> <p> $10 + 4 = 14$  </p>
<p>Vocabulary</p>	<p>share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product</p>		

Year 4

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Divide up to three-digit numbers by a one-digit number using short division.</p>	<p>Use place value counters to divide using the bus stop method alongside.</p> <p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p> 	<p>Children can continue to use drawn diagrams such as the partitioning and dividing using the part part-whole methods learnt in Year 3.</p> 	<p>Children to be taught short division (bus stop). Begin with divisions that divide equally with no remainders.</p>   <p>Children should be made aware that a 0 is used to keep the place value, if the number is not divisible at the start.</p> 
<p>Vocabulary</p>	<p>share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive</p>		

Year 5

Objectives/Strategy	Concrete	Pictorial	Abstract
<p>Divide up to four-digit numbers by a one-digit number using short division.</p>	<p>Use place value counters to divide using the bus stop method alongside.</p> <p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p> 	<p>Children can continue to use drawn diagrams such as the partitioning and dividing using the part part-whole methods learnt in Years 3 and 4.</p> 	<p>Children to use short division (bus stop) method learned in Year 4. Begin with divisions that divide equally with no remainders.</p>  <p>Children should be made aware that a 0 is used to keep the place value, if the number is not divisible at the start.</p> 
<p>Divide at least four-digit numbers by a one-digit number using the short division method and interpreting the remainder appropriately for context.</p>	<p>Use place value counters to divide using the bus stop method alongside with questions which have remainders.</p> <p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p> 	<p>Children can continue to use drawn diagrams such as the partitioning and dividing using the part part-whole methods learnt in Years 3 and 4 with questions which include remainders.</p> 	<p>Continue to use short division (bus stop) method to solve division questions which have remainders.</p> 

Vocabulary

share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Year 6

Objectives/Strategy

Concrete

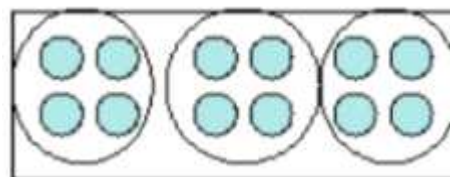
Pictorial

Abstract

Use place value counters to divide using the bus stop method alongside with questions which have remainders.



Children can continue to use drawn diagrams such as the partitioning and dividing using the part part-whole methods learnt in Years 3 and 4 with questions which include remainders.



Children to use short division (bus stop) method learned in Year 4. Begin with divisions that divide equally with no remainders.

$$\begin{array}{r} 312 \\ 3 \overline{)936} \end{array}$$

Children should be made aware that a 0 is used to keep the place value, if the number is not divisible at the start.

$$\begin{array}{r} 073 \\ 3 \overline{)219} \end{array}$$

Divide at least four-digit numbers by a one-digit number using the short division method and interpreting the remainder appropriately for context.

Continue to use short division (bus stop) method to solve division questions which have remainders.

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{)5309} \end{array}$$

Abstract Only

Children to be taught how to divide at least a three-digit number by a two-digit number. See method below.

Divide at least three-digit numbers by a two-digit number using the long division method and interpreting the remainder appropriately for context.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Vocabulary

share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.